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AN ELEMENTARY COURSE

OF

PRACTICAL MATHEMATICS,

FOR THE USE OF SCHOOLS.

PART II.

BY JAMES ELLIOT,

AUTHOR OF "A COMPLETE TREATISE ON PRACTICAL GEOMETRY AND MENSURATION,"
"A PRACTICAL TREATISE ON THE NATURE AND USE OF LOGARITHMS,
AND ON PLANE TRIGONOMETRY," ETC.

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PRACTICAL MATHEMATICS.

PART SECOND.

PRACTICAL GEOMETRY

AND

MENSURATION.

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PRACTICAL GEOMETRY

AND

MENSURATION.

CHAPTER I.

DEFINITIONS AND EXPLANATIONS.

1. The term *Geometry* originally meant the measurement of land: it is now applied to the science which treats of the properties of lines, angles, surfaces, and solids, and to the art of performing certain operations relating to them.

2. *Practical Geometry* merely states those properties and methods of operation, leaving to *Theoretical Geometry* to demonstrate their truth and correctness.

3. The method of determining, in numbers, the lengths of lines, the areas of surfaces, and the contents of solids, is called *Mensuration.*

4. A truth stated is called a *Theorem:* an operation to be performed is called a *Problem.* An *Axiom* is a theorem that is self-evident, and a *Corollary* is an additional statement annexed to a theorem.

5. All material objects are *Solids.*

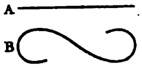
A solid has three dimensions, length, breadth, and thickness.

6. The outside, or boundary of a solid, is called a *Surface* or *Superficies,* which has length and breadth, but no thickness.

7. Regular surfaces are either *Plane,* that is flat, or *Curved,* that is bent.

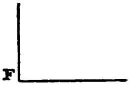
10.* The boundaries of a surface are *Lines*. These have length, but neither breadth nor thickness.

11. Regular lines are either *Straight*, as A, or *Curved*, as B.

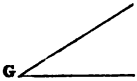


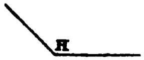
14. The bounds, or ends of a line, are *Points*. These have position or place, but neither length, breadth, nor thickness.

15. An *Angle* is the inclination or opening of two lines which meet in a point, as F, G, H, or I.

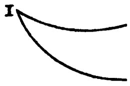


16. When the lines are both straight, it is called a *Rectilineal Angle*, as F, G, or H.





17. When the lines are both curved, it is called a *Curvilineal Angle*, as I.



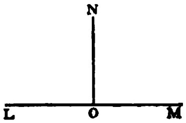
19. The two lines which contain the angle are called its *Sides;* and the point in which they meet, the *Angular Point.*

20. An angle is increased or diminished, by opening or closing the sides, in the same manner as we open or close a knife or a pair of compasses. Thus, the angle F is larger than G, and H large than F. The angle is not altered by lengthening or shortening its sides, and two *Angles* are considered *Equal*, when the sides of the one would coincide with those of the other at the angular points, if the two angles were placed together: if so, they would be equal, howsoever unequal in length the sides of the one might be to those of the other.

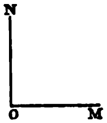
If there is only one angle at the same point, it may be named by the single letter which stands at that point, as F, G, H, or I; but if there are more than one, each angle must be named by three letters, placing that which stands at the angular point in the middle, as the angle LON, or NOL, and the angle NOM or MON.

* In this and the subsequent part of the Elementary Course, the numbers of the paragraphs and propositions are kept the same as in the Author's larger Treatises, in order to enable both works to be used together, when necessary, and that the Key to the latter may serve for the former. This will account for many omitted numbers in this work.

21. One straight line is said to be *perpendicular* to another, when the angles on each side of the former are equal. Thus, NO is perpendicular to LM.

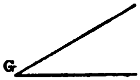


If one part of LM, as LO, were rubbed out, or not drawn, NO would still be said to be perpendicular to the remainder OM, provided that the angles *would be* equal on producing OM to L.

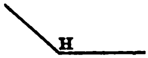


22. The angle on either side of the perpendicular is called a *Right Angle*, as NOL, or NOM, or F.

23. An *Acute Angle* is that which is less than a right angle, as G.

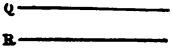


24. An *Obtuse Angle* is greater than a right angle, as H.



25. An *Oblique Angle* is any one that is not a right angle, as G or H: it may be either acute or obtuse.

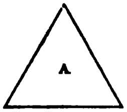
28. *Parallel Lines* are those which are everywhere equally distant from each other, as Q and R.



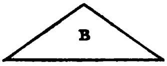
30. *Converging* or *Diverging Lines*, are those which are not parallel to each other.

31. *Oblique* or *Inclined Lines* are those which are neither parallel nor perpendicular.

32. A *Triangle* is a plane surface bounded by three straight lines, as A, B, C, D, or EFG.

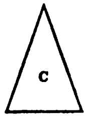


33. An *Equilateral Triangle* is one which has all its sides equal, as A.



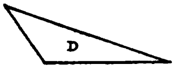
34. An *Isosceles Triangle* has only two of its sides equal, as B or C.

35. A *Scalene Triangle* has no two of its sides equal, as D.



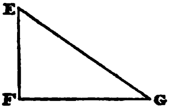
36. An *Acute-angled Triangle* has all its angles acute, as A or C.

37. An *Obtuse-angled Triangle* has one obtuse angle, as B or D.



38. Both obtuse and acute-angled triangles are sometimes called *Oblique-angled Triangles*, as A, B, C, or D.

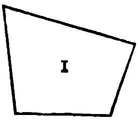
39. A *Right-angled Triangle* is that which has one right angle, as EFG.



40. In a right-angled triangle, the side opposite the right angle is called the *Hypothenuse*, and the other two sides the *Legs*, or the *Base* and *Perpendicular*.

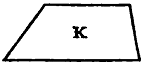
Thus, in the triangle EFG, EG is the hypothenuse, and EF and FG the legs, FG being the base, and EF the perpendicular, or EF the base, and FG the perpendicular, according as we suppose the triangle to be placed.

41. A plane surface, bounded by four straight lines, is called a *Quadrilateral Figure* or *Quadrangle*, as I, K, L, M, N, or O.

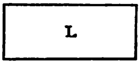


42. If no two of its sides are parallel, it is a *Trapezium*, as I.

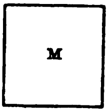
43. If only one pair of its opposite sides are parallel, it is a *Trapezoid*, as K.



44. If both pairs of opposite sides are parallel, it is a *Parallelogram*, as L, M, N, or O.

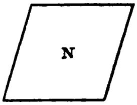


45. A right-angled parallelogram is called a *Rectangle*, as L or M.

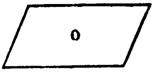


46. A rectangle having all its sides equal, is called a *Square*, as M.

A square has therefore all its sides equal, and all its angles right, and its opposite sides parallel.

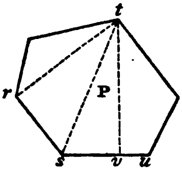


47. An oblique-angled parallelogram having all its sides equal, is called a *Rhombus*, as N.

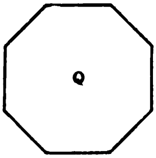


48. When the adjacent sides are not equal, it is a *Rhomboid*, as O.

49. Plane surfaces bounded by more than four straight lines, are called *Polygons*, as P or Q.



50. Of these, a *Pentagon* is a polygon of five sides; a *Hexagon*, of six; a *Heptagon*, of seven; an *Octagon*, of eight (as Q); a *Nonagon*, of nine; a *Decagon*, of ten; an *Undecagon*, of eleven; and a *Dodecagon*, of twelve.



52. A *Regular Polygon* has all its sides and angles equal, as Q. If *either* the sides or the angles are unequal, it is said to be *Irregular*, as P.

54. A *Rectilineal Figure* is that which is bounded by straight lines; and a *Curvilineal Figure*, by curved lines.

55. In any rectilineal figure, as P, a straight line drawn between any two angular points, not next each other, is called a *Diagonal*, as *tr* or *ts*.

56. The *Base* of any figure (plane or solid) is that side on which it is supposed to stand; and its *Altitude* or *Height* is the perpendicular falling on the base from the point which is most distant.

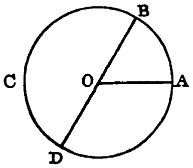
Thus, in the figure P, *su* is taken as the base, and then *tv* is the altitude. In an isosceles triangle the unequal side is usually taken as the base; and in a right-angled triangle, one of the two legs.

57. The *Vertex* of a triangle is the angular point opposite the base.

58. If one extremity of a straight line is fixed, while the line revolves round it on a plain, the surface passed over by the straight line is called a *Circle*.

59. The revolving line (OA) is called the *Radius* of the Circle; the fixed extremity (O) its *Centre*; and the curved line (ABCD) traced out by the other extremity (A), its *Circumference*.

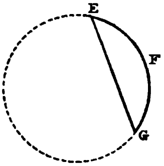
The word *Circle* is sometimes used to denote the circumference instead of the surface within it.



The line which bounds *any* figure is called its *Circumference* or *Perimeter* or *Periphery.*

60. The *Diameter* of a circle is any straight line drawn through the centre, and terminated at both ends by the circumference, as BD.

61. An *Arc* of a circle is any part of its circumference, as EFG.

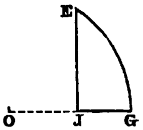


An *arc* of any curve line is any part of the line.

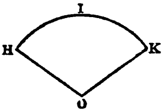
62. A *Chord* is a straight line connecting the extremities of an arc, as EG.

63. A *Segment* of a circle, or of any curvilineal figure, is the space or surface contained between an arc and its chord, as EGF.

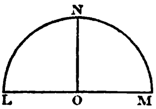
If a radius (OG) be drawn to the one extremity of an arc, and a straight line (EJ) be drawn from the other extremity perpendicular to the radius, the perpendicular (EJ) is called the *Sine* of the arc, and that part (JG) of the radius, intercepted between the arc and the sine, is called the *Versed Sine.*

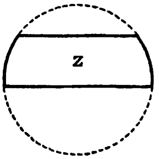


64. A *Sector* of a circle is the space contained between any two radii and their intercepted arc, as HOKI.



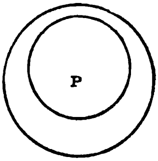
65. When the two radii are in one line, as OL and OM, the sector is half a circle, and is called a *Semicircle*, as LNMO. When the radii are perpendicular to each other, as ON and OM, the sector is one-fourth of a circle, and is called a *Quadrant,* as NOM. A sector which is the sixth part of a circle, is a *Sextant,* and one which is the eighth part, an *Octant.*



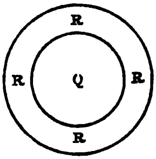


66. A *Zone* of a circle, or of any curvilineal figure, is the space bounded by

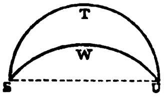
two parallel chords and their intercepted arcs, as Z.



67. When circles have the same centre, they are said to be *Concentric*, as those in the figure Q; when they have separate centres they are said to be *Eccentric*, as in the figure P.

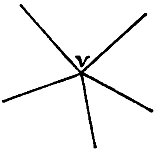


68. A *Circular Ring* is the figure contained between two concentric circles, as RRRR.



69. A *Lune* is the figure formed by two arcs of circles cutting each other, as STUW. The common Chord of the two arcs (SU) is called the *Chord of the Lune.*

70. The Circumference of every Circle is supposed to consist of 360 equal parts, called *Degrees;* a degree to consist of 60 *Minutes;* a minute, of 60 *Seconds;* a second, of 60 *Thirds;* &c.

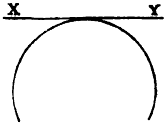


71. If any number of straight lines meet in a common point (V), they form a number of angles occupying all the angular space around that point. The whole of that angular space is supposed likewise to consist of 360 equal parts, called also *Degrees*, which are divided into *Minutes*, *Seconds*, &c., like the degrees of a circle.

Degrees are marked by a small circle over them; minutes, by one accent; seconds, by two, &c.: thus, $6° 4' 14''$ means six degrees, four minutes, fourteen seconds.

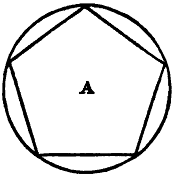
An arc or angle is thus measured by the number of degrees, minutes, &c., which it contains.

It is not, however, the actual length of the arc that is thus determined: a measurement of this kind merely shows what part it is of the whole circumference.

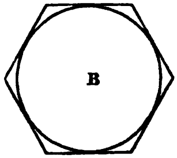


72. A *Tangent* to a circle, or to any other curve, is a straight line touching the curve, but not passing through it at that point, as XY.

73. When all the angular points of a rectilineal figure are in the circumference of a circle (as in figure A) the figure is said to be *Inscribed in the Circle;* or the circle to be *Circumscribed about the Figure.*



74. When all the sides of a rectilineal figure are tangents to a circle (as in figure B) the figure is said to be *Circumscribed about the Circle,* or the circle to be *Inscribed in the Figure.*

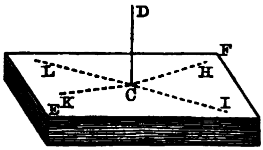


75. *Similar Figures,* whether they be surfaces or solids, are those which are exactly alike in form though they may differ in size, as M and M.

M M

76. Similar plane figures are said to be *similarly placed* when their corresponding sides or other lines are parallel.

77. A *Straight Line* is *Perpendicular* to a *Plane* when it does not lean more towards the plane on one side than on another.



Thus, the line DC is perpendicular to the plane EF, being perpendicular to any straight line (as CI, CH, CK, or CL) which can be drawn from its foot C in the plane EF. Thus, also, a plumb-line suspended over a pool of water is perpendicular to its surface.

78. A *Straight Line* is *Parallel* to a *Plane,* when all points in the line are equally distant from the plane.

Thus, in an ordinary room, the sides or edges of the ceiling, the tops of the doors, the cross-bars of the windows, &c., are parallel to the floor.

79. A *Straight Line* is *Inclined* to a *Plane,* when it is neither parallel nor perpendicular to it.

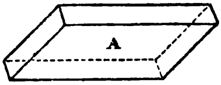
81. *One Plane* is said to be *Perpendicular to another* when it does not lean towards it more on the one side than on the other.

Thus, a wall is perpendicular to the level ground.

82. *Parallel Planes* are those which are everywhere equally distant from each other,

As the floor and ceiling of an ordinary room, or the two sides of a deal.

83. *Converging* or *Diverging Planes* are those which are not parallel to each other; and *Oblique* or *Inclined Planes* are those which are neither parallel nor perpendicular.

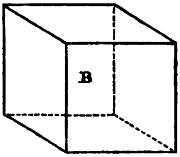


85. A *Parallelopiped* is a solid contained by six planes, every opposite pair of which are parallel, as A.

Its sides are all parallelograms.

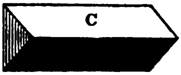
86. When the adjacent sides are perpendicular to each other, it is a *Rectangular Parallelopiped.*

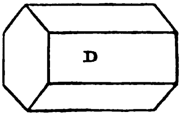
Its sides are then all rectangles.



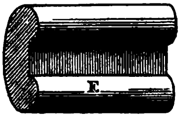
87. When the length, breadth, and height of a rectangular parallelopiped are all equal it is a *Cube*, as B.

Its sides are then all squares.





88. A *Prism* is a solid whose ends are any two parallel plane figures, equal, similar, and similarly placed, and whose sides are determined by straight lines connecting the corresponding points in the circumferences of the two ends, as C, D, or E.



90. When the ends are regular figures it is called a *Regular Prism*, as C and D; when otherwise, *Irregular*, as E.

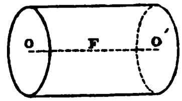
92. When the sides are perpendicular to the ends it is a *Right Prism;* when otherwise, *Oblique.*

The sides of a rectilineal prism are all parallelograms.

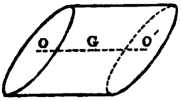
A parallelopiped is also a prism, whose ends are parallelograms, though not usually called so.

93. When the ends of a prism are circles it becomes a *Cylinder*, as F or G.

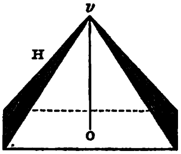
94. The axis of a cylinder is the straight line oo′ connecting the centres of the two circular ends.



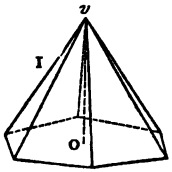
95. A *Right Cylinder* has its axis perpendicular to its ends, as the figure F—like a common drum or roller.



An *Oblique Cylinder* has its axis placed obliquely towards its ends, as the figure G—like a pile of halfpence inclined a little to one side.



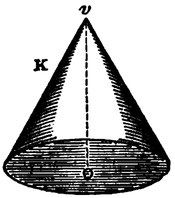
96. A Pyramid is a solid whose base is any plane figure, and whose sides are determined by straight lines connecting the various points in the circumference of the base with a common point out of the base, called the *Vertex*, as H or I, the vertex being *v*.



97. Pyramids are of various kinds, and receive various names according to the forms of their bases, as *Rectilineal*, *Curvilineal*, *Square*, *Pentagonal*, &c. Thus, H represents a square pyramid, such as the famous Pyramids of Egypt are; and I, a hexagonal pyramid.

98. It is called a *Regular Pyramid* when the base is any regular figure, as H and I; when otherwise, *Irregular*.

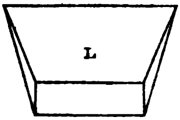
99. When a pyramid has its base circular it becomes a *Cone*, as K.



100. The *Axis* of a regular pyramid or cone is a straight line connecting the vertex with the centre of the base, as *v*O.

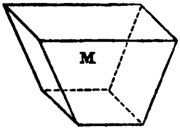
101. When a cone or regular pyramid has its axis perpendicular to its

base it is called a *Right Cone* or *Pyramid*; when otherwise, *Oblique*.



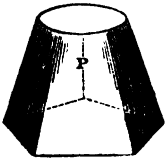
102. A *Wedge* is a solid bounded by five planes, of which the base is a parallelogram; the two sides, trapezoids; and the two ends, triangles, as L.

The edge is, of course, parallel to the base.



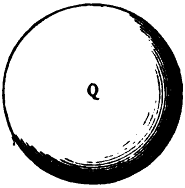
103. A *Prismoid* has, for its ends, any two dissimilar, parallel, plane figures, and has its sides straight, as M and P.

By having its sides straight is meant, that a straight line may be applied at every point in their surface, so as to coincide with the surface from end to end.



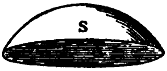
A *Rectangular Prismoid* has its ends both rectangles.

104. A *Sphere* is a round body (like a ball or globe), every point of whose surface is equally distant from a certain point within it called its *Centre*, as Q.

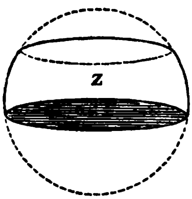


105. The *Radius* and *Diameter* of a sphere are the same as of a circle.

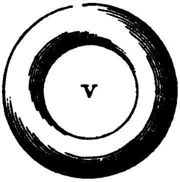
107. A *Segment* of a *Sphere* is a part cut off by any plane passing through the sphere, as S.



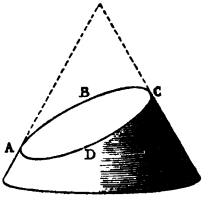
108. If a sphere is cut by two parallel planes, the part intercepted between them is called a *Zone* of the sphere, as Z.



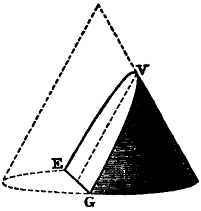
111. A *Cylindric Ring* is a cylinder bent round into the form of a ring, as V.



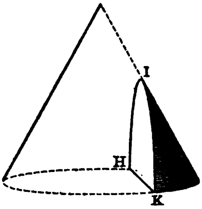
117. If a cone is cut by a plane which passes through both sides of the cone, and is not parallel to its base, the plane figure formed by the section is called an *Ellipse*, as ABCD.



118. If a cone is cut by a plane parallel to one side of it, the section called a *Parabola*, as EVG.



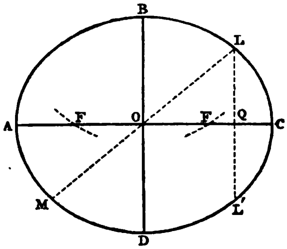
119. If a cone is cut by a plane which intersects only one side and diverges from the other, the section is called a *Hyperbola*, as HIK.



120. These three figures—the Ellipse, the Parabola, and the Hyperbola—are what are called the three *Conic Sections.*

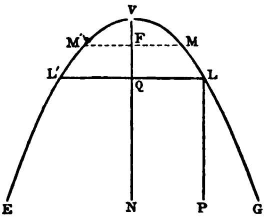
121. The *Vertex* of a parabola or hyperbola is the point nearest the vertex of the cone, as V and I. An ellipse has two vertices, namely, the point C, nearest the vertex of the cone, and the point A, furthest from it.

122. The line (AC) joining the two vertices of an ellipse is called the *Transverse Axis*, or sometimes the *Major Axis*. The middle point (O) of the transverse axis is the centre of the ellipse.

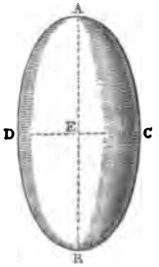


123. Any straight line passing through the centre, and terminated both ways by the circumference, is called a *Diameter*, as LM. The *Conjugate Axis*, or *Minor Axis* (BD), is a diameter perpendicular to the transverse axis.

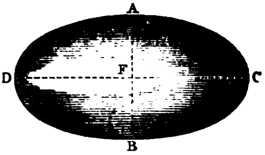
124. A parabola has only one *Axis*, namely, the line (VN) drawn from the vertex within the parabola and parallel to the axis of the cone, or in every point at equal distances from the circumference on both sides, as measured by perpendiculars.



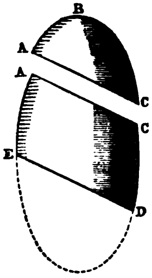
138. If an ellipse (or half ellipse) revolve about either axis (that axis remaining fixed), the solid figure which it describes is called a *Spheroid*, as E or F.



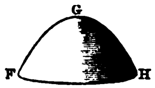
139. It is a *Prolate Spheroid*, when generated by the revolution of the ellipse about its transverse axis, as E; and an *Oblate Spheroid*, when the ellipse revolves about its conjugate axis, as F.



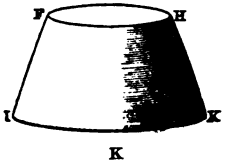
142. A *Segment* of any solid, is a part cut off by a plane passing through the solid, as ABC or FGH.



143. A *Zone*, of any solid with a curved surface, is the part intercepted between two parallel planes, as ACDE.



144. If either of the said planes is what is usually regarded as the base of the solid, the zone is then called a *Frustum*, as FHKI.



CHAPTER II.

PRACTICAL GEOMETRY.

PROPOSITION II. PROBLEM.

To bisect a Straight Line (AB), *that is, to divide it into two equal Parts.*

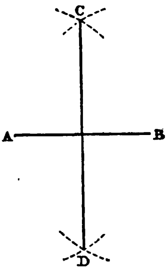
Do it by trials.

NOTE. When it is said, "do it by trials," it is not meant that we are to try and try again, at random, till we attain our object. As much system is required in this as in any other practical process. In the present instance, proceed thus :—Set one foot of the compasses on A, and extend the other to a point C, as near the middle of the line as the eye can judge. Then turn the first foot round towards B, and, if it exactly reach B, the object is accomplished. But if not, make a dot with it at D, and, with the hand, make another dot at a point E, as nearly in the middle of DB as can be determined by the eye. Extend the compasses from C to E, and try if, with this distance, they step exactly twice from A to B. If they do not, repeat the process until the bisection be effected exactly.

PROPOSITION III. PROB.

To draw a Perpendicular through the Middle of a given Straight Line (AB), *not previously bisected.*

With the centres A and B, and a radius equal, or nearly equal, to AB, describe arcs cutting each other in C and D. Draw a straight line from C to D: that line will be the perpendicular required.



EXERCISE. Construct an isosceles triangle having its base 3, and its altitude 2, taken from any scale of equal parts: and determine, by measurement on the same scale, the length of each of the equal sides.*

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* The scholar will perceive that the base is the given line, to be drawn first, and that the altitude is to be set off on the perpendicular to be drawn by the Proposition.

PROPOSITION IV. PROB.

Through a given Point to draw a Straight Line perpendicular to a given Straight Line.

FIRST METHOD.

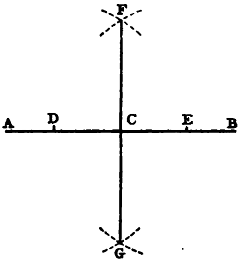
Whether the given Point is in the given Line, or out of it.

Use *a Scale*, or *flat Ruler*, feather-edged on both edges, with a perpendicular line marked across the middle of it. Apply the line so marked to the given line, so that the given point coincide with either edge of the ruler; then draw a line along that edge: that line will be the perpendicular required.

FOURTH METHOD.

When the given Point (C) *is in the given Line* (AB).

With the compasses set off two equal distances, CD and CE, along AB, on each side of C. With the centres D and E, and a radius equal, or nearly so, to DE, describe arcs cutting each other in F and G. Draw the straight line FG; it will be the perpendicular required (being produced if necessary.)

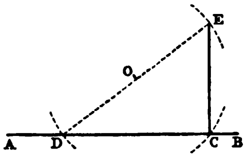


NOTE. When the point is near the end of the given line, the line must be produced before we commence the other operations. When that is impossible, proceed by the following method.

FIFTH METHOD.

When the given Point (C) *is in the given Line* (AB), *near its extremity* (B), *and the Line cannot be produced.*

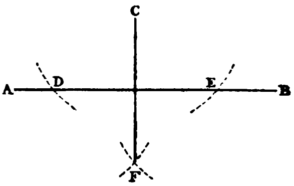
Take a point, O, out of the line, and on the same side of C on which A lies. With the centre O, and radius OC, describe a circle cutting AB in D. Through D draw DOE, cutting the circle in E. Draw EC, which will be the perpendicular required.



SIXTH METHOD.

When the Point (C) *is out of the Line* (AB.)

With the centre C, and as long a radius as can be used conveniently, cut AB in D and E. With the centres D and E, and a radius greater than half DE, describe arcs intersecting in F. CF, when drawn, will be the perpendicular required.

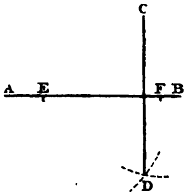


NOTE. If C be nearly opposite the end of AB let AB be produced, if possible; if not, proceed by the following method.

SEVENTH METHOD.

When the Point (C) *is out of the Line* (AB), *nearly opposite the End of it, and when the Line cannot be produced.*

In AB take two points, E and F, as near to A and B as convenient. With these points as centres, and with radii reaching to C, describe arcs on the other side of the line, intersecting each other in the point D. CD, when drawn, will be the perpendicular required.



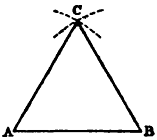
EXERCISE 1. Construct a right-angled triangle, having one of its legs 2 and the other 3, taken from any scale of equal parts.

2. Construct a right-angled triangle, having its hypothenuse 5, and one of its legs 4, and determine, by measurement, what will be the length of the other leg.

PROPOSITION V. PROB.

On a given Straight Line (AB) *to construct an Equilateral Triangle.*

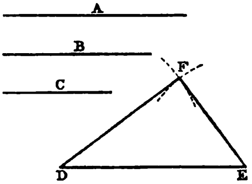
With the centres A and B, and radius AB, describe arcs cutting each other in C. Draw AC and BC.



PROPOSITION VI. PROB.

To construct a Triangle having its three Sides equal to three given Straight Lines (A, B, and C.)

Draw a straight line, DE, equal to one of the given straight lines, A. With the centres D and E, and radii equal to B and C, describe arcs intersecting in F. Draw FD and FE. FDE will be the triangle required.



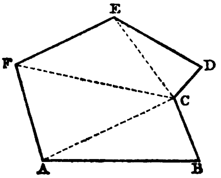
EXERCISE 1. Construct a triangle having its three sides respectively $1\frac{1}{2}$, 2, and $2\frac{1}{2}$ inches in length.

2. Construct an isosceles triangle having its base 2, and each of its equal sides 3.

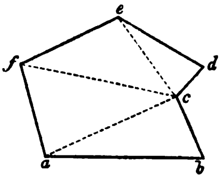
PROPOSITION VII. PROB.

To construct a Rectilineal Figure equal and similar to another given Rectilineal Figure (ABCDEF).

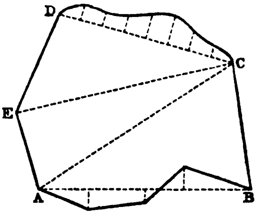
Divide the given figure into triangles by drawing diagonals. Make a triangle *abc* equal to the triangle ABC (by the last Proposition), then the triangle *acf* equal to ACF; and so on till the figure is completed.



NOTE 1. The figure may be divided into triangles in various ways; but that division is to be preferred which produces the fewest, and those the most nearly equilateral.



NOTE 3. If the outline, or any part of it, consist of a great number of minute portions, as along the side AB of the annexed figure, it will be best to draw one or more imaginary straight lines, as AB, as near the irregular outline as possible, and to draw perpendiculars to those



lines from the various angular points opposite them. After the space within the imaginary lines has been divided into triangles, and copied, as before, the positions of the perpendiculars upon the imaginary lines must be measured, and laid down on the new plan; and from the points thus marked, perpendiculars must be drawn equal to those on the given figure. In this way, also, a curved boundary, as that upon the side CD, may be copied with any degree of accuracy. In the latter case, the work is facilitated by setting off the perpendiculars at equal distances. (See Prop. LXXVI and Notes.)

Exercise 1. Copy exactly the trapezium, I, and the polygon, P, shown in the Definitions of plane figures.

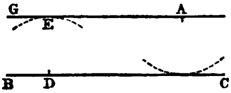
2. Construct a figure equal and similar to that in the last diagram of this Proposition.

Proposition VIII. Prob.

Through a given Point (A) *to draw a Straight Line parallel to a given Straight Line* (BC).

FIRST METHOD.

With the Parallel Ruler.[*]



Lay the ruler, closed, with the outer edge of the one bar along BC, and on the contrary side to that on which A lies. Press the fingers of the left hand firm on the other bar, and open the first bar until its outer edge touch the point A. Remove the fingers from the second bar, and press the first firmly until a line is drawn along the edge which touches A.

Note. The one bar has the feather edge flat upon the paper; the other has it raised above it. The former is intended to be used with pencil, the latter with ink.

SECOND METHOD.

With the Compasses.

Place the one foot of the compasses in A, and so adjust the other, that, when swept round, it may just touch the line BC. (See the preceding diagram.) In BC take any point D, as remote from A as convenient. With the centre D, and the same distance, as before, in the compasses, describe an arc of a circle, E. Draw a straight line, AG, passing through A, and just touching the arc E.

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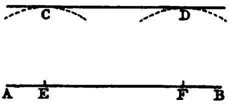
[*] The common parallel ruler is here alluded to. There is another form, of later invention, consisting of a flat ruler rolling on two wheels or on a cylinder. The use of the latter instrument will be understood as soon as it is handled.

PROPOSITON IX. PROB.

To draw a Straight Line parallel to a given Straight Line (AB), *and at a given Distance from it.*

In AB take any two points E and F, near to A and B. With the centres E and F, and the given distance in the compasses, describe two arcs, C and D, opposite E and F, and draw the straight line CD, touching both arcs.



NOTE. The given distance may either be a given straight line, or a given number of parts on a known scale, as inches for instance. In the latter case, the distance must be taken from the scale with the compasses.

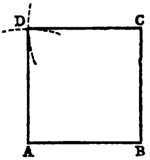
EXERCISE 1. Draw a straight line, and another parallel to it, at the distance of $\frac{7}{10}$ of an inch.

2. Construct any parallelogram, having its opposite sides distant respectively 1 inch and $\frac{1}{4}$ of an inch from each other.

PROPOSITION X. PROB.

To construct a Square on a given Straight Line (AB).

Draw BC perpendicular and equal to AB. With the centres A and C, and radius AB, describe arcs intersecting in D. Draw AD and CD.



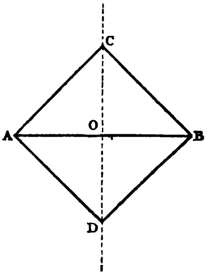
NOTE. If the work is intended to be very correct, it should be proved, when the pencilling is finished, by measuring if the diagonals are equal.

EXERCISE. Construct a square having its side $\frac{3}{4}$ of an inch.

PROPOSITION XI. PROB.

To construct a Square on a given Diagonal (AB).

Bisect AB in O by the perpendicular CD (by Prop. III). Make OC and OD each equal to OA or OB. Draw AC, BC, BD, and AD.

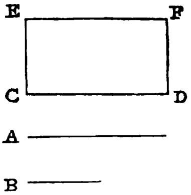


EXERCISE. Construct a square having its diagonal 1 inch.

PROPOSITION XII. PROB.

To construct a Rectangle having its Length and Breadth respectively equal to two given Lines (A and B).

Draw CD equal to A; and, perpendicular to it, draw EC and FD, each equal to B. Join EF.



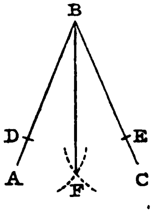
NOTE. The correctness of the work may be verified as in the case of the square.

EXERCISE. Construct a rectangle having its length 1 inch, and its breadth $\frac{3}{4}$.

PROPOSITION XIII. PROB.

To bisect a given Angle (ABC), *that is, to divide it into two equal Angles.*

On BA and BC set off two equal distances, BD and BE, as long as they can conveniently be taken. With the centres D and E, and any sufficient radius not greater than DE, describe arcs intersecting in F. Draw BF, which will divide the given angle into two equal angles, ABF and CBF.

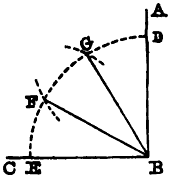


Exercise. Construct an isosceles triangle having a right angle at its vertex, and its altitude 1 inch.

Proposition XIV. Prob.

To trisect a Right Angle (ABC), *that is to divide it into three equal Angles.*

With the centre B, and radius as large as convenient, describe the arc ED. With the same radius, and the centres D and E, intersect the former arc in F and G. Draw BF and BG.

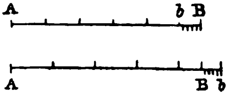


Corollary. In the same manner we may trisect an arc or a sector of 90°.

Proposition XV. Prob.

To divide a given Straight Line (AB) *into any given Number of equal Parts* (*say five*).

Do it by trials, proceeding thus:—Take, in the compasses, as nearly one-fifth of the given line as the eye can judge, and with this make five steps along the line from A towards B. If the last step reach B exactly, the work is done. But if it either come short of B (as in the first figure), or reach beyond it (as in the second figure), the last step falling on *b*, in either case take a fine pencil, or steel point, and divide the small portion B*b* into five parts, as nearly equal as they can be made by the eye: then lengthen or shorten the distance previously in the compasses by one of the smaller divisions, and with this new distance step again along AB; and, if it still fails, repeat the process till successful.



Note 2. Until the learner has acquired some practice in dividing, he ought to transfer the given line to a separate slip of paper, and, having divided it there, to re-transfer the divisions to the given line; otherwise, he will probably

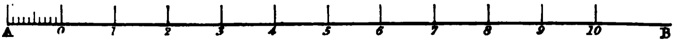
disfigure, with repeated marks, the drawing he may have in hand.

Note 3. If the number of parts into which the line is to be divided is not a prime number, resolve it into its prime factors. Thus, if it were required to divide the line into 30 equal parts, since $30=2\times3\times5$, first divide it into 5 parts, then each of these into 3, and again, each of the last into 2.

Exercise. Draw a line of 5 inches, and divide it into six equal parts.

Proposition XVI. Prob.

To use a Plane Decimal Scale of equal Parts.



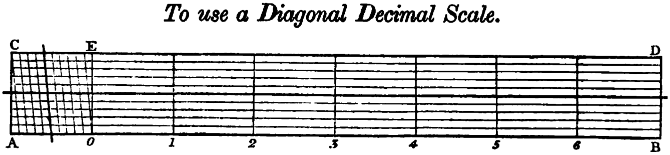
To take any number from the scale, as 36. Put the one foot of the compasses to 3 on the longer divisions, and extend the other to the sixth of the smaller divisions, counting from 0.

To measure any line upon the scale. Take its length in the compasses, and observe the number of greater and smaller divisions which it takes in on the scale.

Exercises. The teacher may see the scholar take the following numbers from the scale, and measure the lengths of one or two given lines: 26, 84, 17, 65;* 650; 6500; 6·5, ·65; ·065, &c.

Proposition XVII. Prob.

To use a Diagonal Decimal Scale.



To take any number from the scale, as 365. For 360 extend the compasses (as in the last problem) from the point marked 3, between 0 and B, to the 6 mark between 0 and A; but this is too little: therefore, for the 5, move both feet of the compasses along the two cross lines, which they now point to, till they come to the fifth longitudinal line from AB. In doing this, it will be observed, that they require to be opened a very small way: this additional minute distance is for the 5 in the unit's place. As explained in the last problem, the larger divisions may represent feet, yards, miles, units, tens, hundreds, or any equal lengths, provided that the next smaller represent tenth parts of these, and that the smallest divisions of all (made by the oblique lines) represent tenth parts of the latter; and remembering that the same parts must always represent the same lengths in the same diagram, drawing, or plan.

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* The semicolon here indicates a change of scale, or rather the transference of the same scale to a higher or a lower place of decimals.

How to measure any line upon the scale will be obvious, from the reverse process.

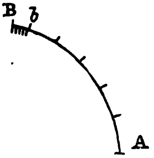
Exercise 1. The teacher may direct the younger scholars how to take the following numbers from the scale, and how to measure the lengths of one or two given lines: 260, 261, 265, 735, 486, 698, 307, 592; 59·2; 5·92; ·592, 0·592; 5920.

Exercise 2. Construct a triangle having its three sides respectively 500, 480, and 374 links, or 5, 4·8, and 3·74 chains.

Proposition XVIII. Prob.

To divide an Arc of a Circle (AB) *into any given Number of equal Parts.*

Proceed by the same method as that for dividing a straight line in Problem xv, observing the three Notes.



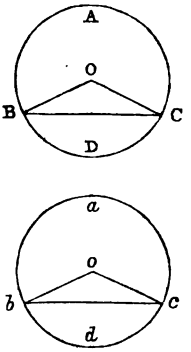
Note. When the division has been brought very near the truth, but still exceeds or falls short by a very minute quantity, it is better (if the hair divider is not used), instead of altering the compasses again, to step on the inner margin of the arc, or on the outer, according as the distance in the compasses is too short or too long.

PROPOSITION XIX. THEOREM.

In equal Circles equal Arcs have equal Chords and equal Angles at the Centre.

For instance, in the two equal circles ABC and *abc*, let the arcs BDC and *bdc* be equal, then their chords BC and *bc* will also be equal, and the angle BOC will be equal to the angle *boc*, O and *o* being the centres.

COROLLARY. The converse of this proposition is also true, that—*In equal circles equal chords have equal arcs, and,* consequently, *equal angles at the centre;* and that, *Equal angles at the centre have equal arcs, and,* consequently, *equal chords.*

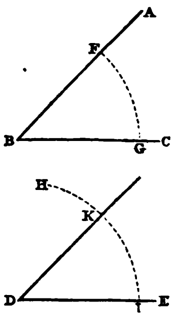


NOTE. On this proposition, and the corollary, depend the solutions of the two following problems, as well as of many others.

PROPOSITION XX. PROB.

At a given Point (D), *on a given Straight Line* (DE), *to make an Angle equal to a given Angle* (ABC).

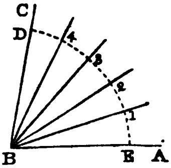
With the centres B and D, and as large a radius as convenient, describe the arcs FG and HI. Take the distance FG in the compasses, and set it off on the other arc, from I to K. Draw DK. EDK will be the angle required.



PROPOSITION XXI. PROB.

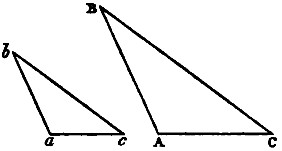
To divide an Angle (ABC) *into any given Number of equal Angles.*

With the centre B, and as long a radius as convenient, describe the arc DE. Divide that arc into the requisite number of equal parts (by Prop. XVIII), in the points 1, 2, 3, &c., and draw the straight lines B 1, B 2, B 3, &c.*



PROPOSITION XXII. PROB.

To construct a Triangle similar to a given Triangle (ABC), *on a given Straight Line* (*ac*), *corresponding to a given Side* (AC) *of the given Triangle.*



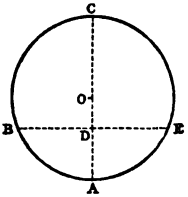
Make the angles *bac* and *bca* equal, respectively, to the angles BAC and BCA.

EXERCISE. On a base of 1 inch construct a triangle similar to either of the two triangles in the diagram.

PROPOSITION XXIII. PROB.

Having given a Circle, of which the Centre is not known, it is required to find the Centre.

FIRST METHOD.



Draw any chord, BE, and bisect it in D by the perpendicular AC (by Prop. III). Bisect AC in O (by Prop. II). O will be the centre.

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* No general rule can be given, correct in theory, for dividing either an arc or an angle into *any* proposed number of equal parts, by means of straight lines and circles only. Mathematicians long attempted to discover such a rule, but in vain; all that we can do is, to divide it, by approximation, so near the truth, that the error shall be less than the eye can detect.

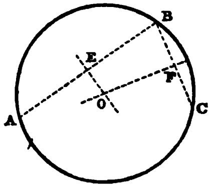
SECOND METHOD.

Find it by trials.

Exercise. Take any round object, as a penny piece: lay it on paper: trace the circumference with a pencil: find the centre: with that centre describe a circle with the compasses, and observe if it agrees with the other circle.

Proposition XXIV. Prob.

To describe the Circumference of a Circle through any three given Points (ABC) *which are not in one Straight Line.*



Draw BA, BC, and bisect them by the perpendiculars EO, FO, meeting in O. With the centre O, and distance OA, describe a circle, which will also pass through the two other points, B and C.

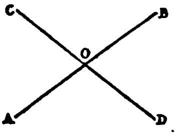
Exercise 1. Three points cannot be placed so irregularly (no matter how) that a circle may not be made to pass through them, unless they are all in one straight line. The learner will probably have some doubt on the subject: let him try if he can place them so that the rule shall not answer.

2. The distances of three points from each other are, 2, 3, and 4. Place them on the paper (by Prop. VI), and make a circle pass through them.

3. The chord of an arc is 5, and the height 2: describe the arc.

Proposition XXV. Theorem.

If two Straight Lines cut each other, the vertical or opposite Angles are equal.



That is (referring to the diagram), the angle AOC is equal to the angle BOD, and AOD equal to BOC.

Exercise. Draw any two lines intersecting each other, and try if the opposite angles are equal by Prop. XIX or XX.

Proposition XXVI. Theorem.

All right Angles are equal.

Proposition XXVII. Theorem.

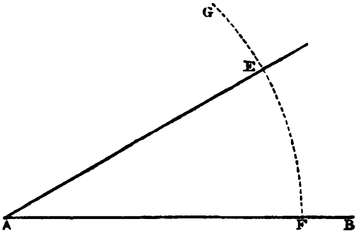
Every right Angle contains 90°.

Proposition XXIX. Problem.

At a given Point (A), *and on a given Straight Line* (AB), *to make an Angle of any proposed Number of Degrees* (*say* 28).

FIRST METHOD.

With a Scale of Chords.



Extend the compasses from 0 (or C) to 60, upon a scale of chords. With this radius, and the centre A, describe the arc FG. Again, extend the compasses from 0 to 28, on the same scale, and set it off on the arc from F to E. Draw AE. BAE will be the angle required.

Note 1. The same radius (the chord of 60°) with which we describe the arc is to be taken for all angles.

Note 2. If the angle is of more than 90°, set it off in two parts. Thus, for an angle of 110°, set off the chord of 60°, then the chord of 50°, continuously upon the same arc. It is always best to take the chord of 60° for the first part, or, if the angle is of more than 120°, to take the chord of 60° twice, and then that of the remainder.

SECOND METHOD.

With a Protractor.

Apply the central point of the protractor to the point A, bringing the mark of 28° to AB. Then draw a line along that side of the protractor in which the centre lies.

Or, apply the central point of the protractor to the point A, laying the straight edge, or the line marking 0°, along AB. Make a dot, E, at the other edge or circumference of the protractor, at the division answering to the number of degrees proposed (28°). Then draw the line AE.

Or, lay the line marking 90° upon AB, and count the degrees from it.

NOTE. Protractors are always to be found, in complete cases of instruments, either in a semicircular form, made of brass, or on a broad and thin ivory scale. On both there is a central point on one side, from which the lines marking the several divisions radiate; and round the other sides, or the circumference, the degrees are marked on the edge.

EXERCISE 1. Make an angle of 45°, an angle of 95°; another of 120°, and a fourth of 170°.

2. Make a triangle having its base 2 inches, and the two angles at the base 30° and 50°.

3. Construct a triangle having two of its sides $1\frac{1}{2}$ and 2 inches, and the angle contained between them 80°.

4. Make a parallelogram having its two adjacent sides 1 and 2 inches, and their included angle 100°.

PROPOSITION XXX. PROBLEM.

To measure the Number of Degrees which any given Angle (EAF) *contains.* (*See the last Diagram*).

FIRST METHOD.

With the Scale of Chords.

With the centre A, and a radius equal to the chord of 60° on the scale, describe the arc EF, or rather, simply mark the points E and F. Extend the compasses from E to F. This extent, applied to the scale of chords, will stretch from C (or 0) to the figure which indicates the number of degrees contained in the given angle.

NOTE. If the angle is of more than 90°, measure it in two or more parts, beginning always with the chord of 60°, &c., as directed in Note 2 to the last problem.

SECOND METHOD.

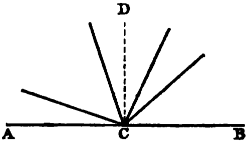
With the Protractor.

Apply the undivided edge to one side of the angle, placing the central point of the protractor upon the angular point. The other side of the angle will then point out, on the edge of the protractor, the number of degrees which the angle contains.

EXERCISE. What is the number of degrees in each of the four angles in the diagram to Proposition xxv ?

PROPOSITION XXXIII. THEOREM.

All the Angles formed by a Straight Line (AB) *and any Number of other Lines meeting in the same Point* (C) *and all on the same Side of the first Line, are together equal to two Right Angles, and contain* 180 *degrees.*

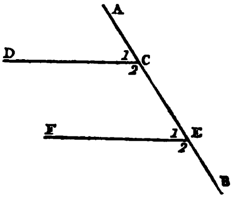


PROPOSITION XXXIV. THEOREM.

If from any two Points in a Straight Line (AB) *two other Straight Lines* (CD and EF) *be drawn parallel to each other, these two make equal Angles with the first Line.*

Thus, in the diagram, the angles marked 1 are equal to each other. So also the angles marked 2 are equal to each other, but not to the other two.

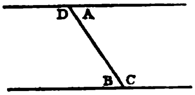
If the parallel lines were drawn *through* the other line, they would also make two pairs of equal angles on the other side of it.

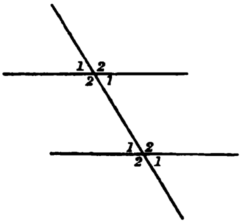


PROPOSITION XXXV. THEOREM.

If a Straight Line meet two parallel Straight Lines, it makes the alternate Angles equal to each other.

Thus, the angle A is said to be alternate to B, and is equal to it; so also is C to D.

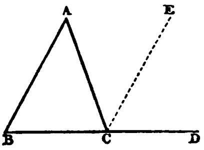




NOTE. From this and the preceding propositions it will appear, that all the angles marked 1 in the annexed diagram are equal to each other, and also, that all the angles marked 2 are equal to each other.

PROPOSITION XXXVI. THEOREM.

If one side of a Triangle (ABC) *be produced, the exterior Angle* (ACD) *will be equal to the two interior and opposite Angles* (A and B) *together.*



EXERCISE 1. If the two angles A and B (in the diagram) contain, respectively, 35° 20′ and 72° 50′, how many degrees are in the angle ACD?

2. If A be an angle of 43° 6′, and ACD of 108° 44′, what is the angle B?

PROPOSITION XXXVII. THEOREM.

The three Angles of any Triangle are together equal to two Right Angles, and contain 180 *Degrees.*

EXERCISE 1. Two angles of a certain triangle contain, respectively, 80 and 90 degrees: how many degrees are in the third?

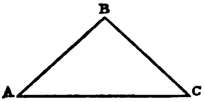
2. One of the angles of a right-angled triangle is of $27\frac{1}{4}$ degrees; what is the other acute angle?

3. Construct a right-angled triangle having one of its sides 2 inches, and the angle opposite of 35°.

PROPOSITION XXXVIII. THEOREM.

The Angles at the Base of an Isosceles Triangle are equal.

Let ABC be an isosceles triangle, having the sides BA, BC, equal: then the angles A and C are also equal.



COROLLARY. *Hence, the three angles of an equilateral triangle are all equal, and contain each* 60 *degrees, and from this it follows that the chord of* 60° *is equal to the radius.*

EXERCISE 1. When one of the angles at the base of an isosceles triangle is 70°, what is the angle at the vertex?

2. If an isosceles triangle is right-angled at the vertex, how many degrees are in each angle at the base?

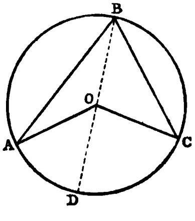
3. Draw an isosceles triangle having the angle at the vertex 30°, and the base 1 inch.

PROPOSITION XXXIX. THEOREM.

Two Triangles are equal in every Respect, if the three Sides of the one are equal to the three Sides of the other; or, if two Sides and their included Angle of the one are equal to two Sides and their included Angle of the other; or, if two Angles and a Side of the one are equal to two Angles and the corresponding Side of the other.

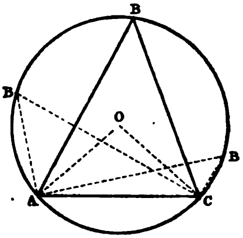
PROPOSITION XL. THEOREM.

The Angle (AOC) *at the Centre of a Circle is double of the Angle* (ABC) *at the Circumference, when both stand on the same Arc* (AC).



Proposition XLI. Theorem.

The Angles in the same Segment of a Circle are equal.

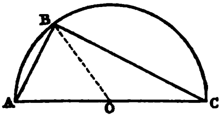


If any point B be taken in the arc of a segment ABC, and BA BC be drawn, the angle ABC is said to be an *angle in the segment*. If B be taken at any other point in the arc, the angle ABC is always of the same magnitude in the same segment.

Exercise. Describe a segment of a circle, take different points in its arc, join those points with the extremities of the arc, as in the diagram above, and try if the angles at all of those points are equal (by Prop. xix, Cor.).

Proposition XLIII. Theorem.

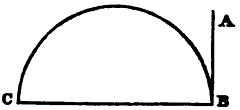
The Angle in a Semicircle is a Right Angle.



Let ABC be a semicircle; any angle in it, as ABC, is a right angle, no matter in what part of the circumference the point B may be placed.

Proposition XLIV. Theorem.

A Straight Line perpendicular to the Diameter of a Circle at its Extremity, is a Tangent to the Circle.



Thus, AB, being perpendicular to the diameter BC, is a tangent to the circle at B.

Exercise. Describe a circle: take a point in the circumference, and draw a tangent to the circle at that point.

PROPOSITION XLV. PROB.

To draw a Tangent to a Circle from a given Point (A) *without it.*

(See the last diagram.)

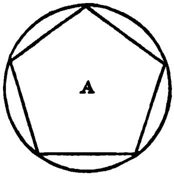
Apply the edge of a ruler so as to touch both the given point and the circumference, and draw a line along it. If it is also required to know the exact point in which the tangent touches the circle, draw a perpendicular to it from the centre.

Exercise. Describe a circle: draw a tangent to it anywhere with the ruler: it is required to determine the exact point in which it touches the circle.

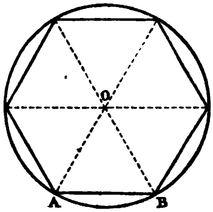
PROPOSITION XLVI. PROB.

In a given Circle to inscribe an Equilateral Triangle, a Square, or a regular Polygon of any given Number of Sides.

Divide the circumference into the same number of equal parts as the number of sides of the proposed polygon (by Prop. XVIII, observing the Note), and draw straight lines connecting the several points of division.



For the Square, the circumference may be divided by drawing two diameters perpendicular to each other, or by drawing one diameter, and bisecting each semi-circumference.



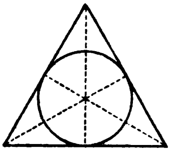
For the Hexagon, take the radius in the compasses, and with that distance step round the circumference. This will divide it at once into six equal parts.

For the Equilateral Triangle, divide the circumference into six equal parts, in the same way as for the hexagon, and join the alternate points of division.

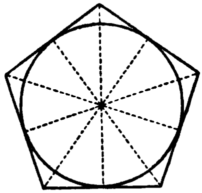
Exercise. Describe five circles, and inscribe in the first a square, in the second an equilateral triangle, in the third a dodecagon, in the fourth a nonagon, in the fifth a heptagon.

Proposition XLVII. Prob.

About a given Circle to circumscribe an Equilateral Triangle, Square, or regular Polygon of any given Number of Sides.



Divide the circumference into the given number of equal parts (by Prop. xviii): draw radii to the several points of division; and, through the same points, draw perpendiculars to the several radii.



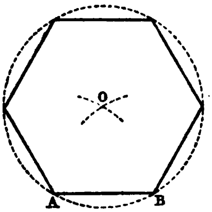
For the Equilateral Triangle, the Square, and the Hexagon, the circumference may be divided by the particular methods shown in the last Proposition.

Note. When the polygon is of an odd number of sides, the radii, when produced, should pass through the angular points of the polygon, as in the figure shown above. This serves as a good check on the accuracy of the operation.

Proposition XLIX. Prob.

On a given Straight Line (AB) *to construct a regular Hexagon.*

With the centres A and B, and radius AB, describe arcs intersecting in O. With the centre O, and radius OA or OB, describe a circle. With AB in the compasses, step round the circle and join the adjacent points.

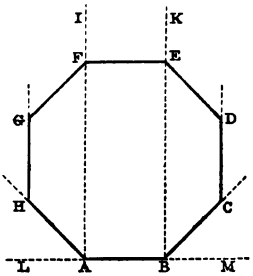


Exercise. Draw the figure of a piece of honeycomb, showing seven or more contiguous cells.

Proposition L. Prob.

On a given Straight Line (AB) *to construct a regular Octagon.*

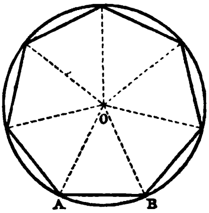
Produce AB to L and M, and, perpendicular to it, draw AI and BK; and, before proceeding further, ascertain that these two lines are exactly parallel. Bisect the angles IAL and KBM, and upon the bisecting lines set off AH and BC, each equal to AB. Take the distance HC in the compasses, and set it off on AI and BK, to F and E. From H and C draw HG and CD parallel to AI or BK, and equal, each, to AB. Try if GF and ED are each equal to AB. Draw GF, FE, and ED.



PROPOSITION LI. PROB.

On a given Straight Line (AB) *to make a Polygon of any given Number of Sides.*

Divide 360° by the given number of sides: this will give the angle AOB at the centre. Subtract the quotient, just obtained, from 180°. Make the angles OAB and OBA each equal to half the number of degrees last found. With the point of intersection O as a centre, and radius OA or OB, describe a circle; and with the distance AB step round the circumference, and join the adjacent points of division.



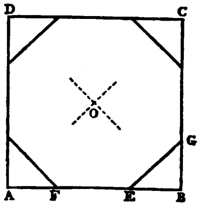
NOTE. Since OA and OB bisect the two angles of the polygon, one of the angles of the polygon will be equal to the sum of OAB and OBA; that is, after subtracting AOB from 180°, the remainder is an angle of the polygon. In Table I, at the end of this volume, will be found the angle at the centre, the angle of the polygon, and the half angle of the polygon (OAB or OBA), ready calculated, for any polygon of not more than twelve sides.

EXERCISE. On a straight line of one inch construct a regular decagon.

PROPOSITION LII. PROB.

To cut off the Corners from a given Square (ABCD), *so as to form an Octagon.*

Draw the two diagonals; or simply mark where they cut each other, in O. Set off the distance AO from A to E, and from B to F; and the same on each of the other sides of the square. Connect the points thus found by straight lines.



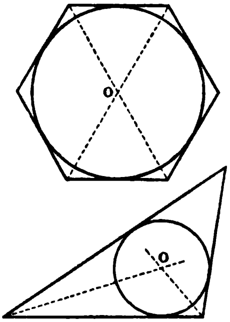
EXERCISE. Construct an octagon which shall have the distance between its opposite sides 2 inches.

PROPOSITION LIII. PROB.

In any given Triangle, Square, or regular Polygon, to inscribe a Circle.

Bisect any two or more of the angles by two straight lines meeting in a point O. With O as a centre, and a radius sufficient to reach any one of the sides without cutting it, describe a circle. That circle will touch all the sides.

NOTE. In the case of the square, or of a polygon of any even number of sides, the angles are most easily bisected by drawing straight lines between the opposite angular points (as in the figure for the hexagon).

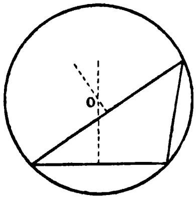


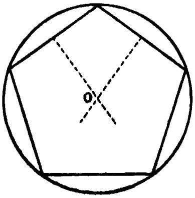
PROPOSITION LIV. PROB.

About any given Triangle, Square, or regular Polygon, to circumscribe a Circle.

Bisect any two or more of the sides by perpendiculars meeting in O. With the centre O, and a radius sufficient to reach any of the angular points, describe a circle which will pass through them all.

NOTE. If the polygon is of an even number of sides, the centre may be found by joining any two opposite angular points.





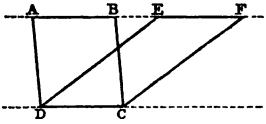
EXERCISE 1. On a line of 2 inches construct a square. Cut off its corners so as to form a regular octagon. Inscribe a circle in the octagon, and describe another about it.

2. Construct a triangle, having its sides 2, 3, and 4, from any scale: inscribe a circle in it, and circumscribe a circle about it.

Proposition LVI. Theorem.

Parallelograms on the same Base and between the same Parallels are equal.

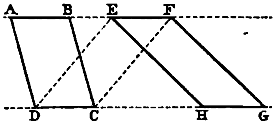
Let the two parallelograms ABCD and CDEF stand on the same base CD, and have their sides opposite CD both in a line, AF, parallel to CD; then the one parallelogram is equal to the other.



Proposition LVII. Theorem.

Parallelograms on equal Bases and between the same Parallels are equal.

The two parallelograms ABCD and EFGH, on equal bases, CD and GH, and between the same parallels, AF and DG, are equal to each other.



Corollary. *Parallelograms of equal bases and of equal altitudes are equal.*

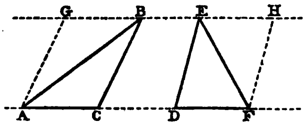
Exercise 1. Draw any parallelogram, and make a rectangle equal to it.

2. Divide the rectangle just made into three equal rectangles.

Proposition LVIII. Theorem.

Triangles on equal Bases and between the same Parallels are equal.

The two triangles ABC and DEF, on equal bases AC and DF, and between the same parallels, are equal.



EXERCISE 1. Construct any equilateral triangle, and make a right-angled triangle equal to it.

2. Construct a triangle, having its sides 5, 7, 10, and then make an isosceles triangle equal to it.

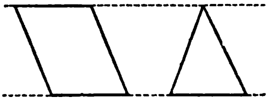
3. Divide the isosceles triangle just made into five equal triangles.

4. The side of an equilateral triangle being 1 inch, make a triangle equal to it, having an angle of 75° at the base.

5. The base of a given right-angled triangle is 1, and the perpendicular 3: make a triangle equal to four times the given triangle.

PROPOSITION LIX. THEOREM.

If a Parallelogram and a Triangle are upon equal Bases, and between the same Parallels, the Parallelogram is double of the Triangle.



EXERCISE 1. Draw any triangle, and make a parallelogram equal to it.

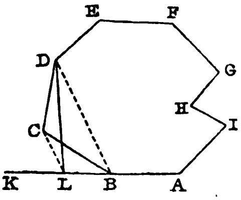
2. Make a triangle whose area shall be equal to a square inch.

3. The sides of a triangle are 2, 3, and 4: make a rect angle equal to it.

PROPOSITION LXII. PROB.

Having given any Rectilineal Figure (ABCDEFGHI), *to change it into an equal Rectilineal Figure, having one Side fewer.*

Produce any side as AB to K. Join BD, and parallel to it draw CL. Join DL. Then ALDE FGHI will be the required figure, the two sides, AL, LD supplying the place of the three sides AB, BC, CD.



Note. If we use a parallel ruler the lines BD and CL need not be drawn: it is sufficient to lay the edge of the ruler to the points B and D, then slide it to C and mark the point L where it crosses AK. The observance of this remark will be of great service in performing operations under the next proposition, especially where the sides of the figure are numerous, since the multiplicity of lines required would otherwise become perplexing.

Exercise 1. Draw any trapezium, and change it into an equal triangle.

2. Construct a regular hexagon, and convert it into an irregular pentagon of equal area.

Proposition LXIII. Prob.

To change any given Rectilineal Figure into a Triangle of equal Area.

Change it into an equal figure of one side fewer (by the last proposition), and repeat the process till only three sides remain.

Note 1. A part of the work may be performed on the one side of the figure, and the rest on the other; or, we may even go on with the work at several of the angular points if preferred. After a little practice it will be observed that not a single line is actually required to be drawn in the operation except the producing of one or more sides, and the three lines forming the required triangle.

Note 2. It will be observed that, by the same method, we may substitute a straight line for any crooked part in the boundary of a figure without altering its area. This is what is called *rectifying a crooked boundary.*

Exercise 1. Construct a regular pentagon, and convert it into an equal triangle.

2. The same with a regular heptagon.

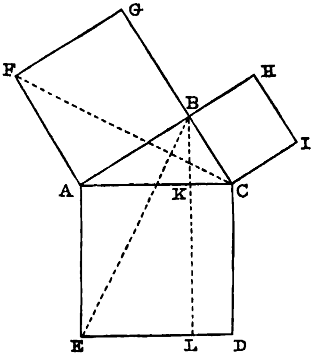
3. Delineate a figure equal and similar to that in the last proposition (by Prop. vii), and change it into an equal triangle.

4. Draw a rectangle equal to any regular hexagon previously constructed.

PROPOSITION LXVI. THEOREM.

The Square described on the Hypothenuse of a right-angled Triangle is equal to the Sum of the Squares described on the two other Sides.

Let ABC be a right-angled triangle, and let AD be a square described on the hypothenuse, and AG and CH squares described on the two sides. AD will be = the sum of AG and CH.



COROLLARY 1. Hence, *the square on one of the sides is equal to the difference of the squares on the hypothenuse, and on the other side.*

EXERCISE 1. There are two squares whose sides are 30 and 40; make a square equal to both of them, and say what is the length of its side.

2. Make a square equal to the difference of the same two squares.

3. There are three squares whose sides measure 12, 15, and 16 equal parts: make a square equal to the sum of the three, and state the length of its side.

4. Make a square equal to four times a square whose side is 3.

5. Make a square equal to seven times a square whose side is 100, and say what is the length of its side.

The answer will be 265 nearly.

COROLLARY 2. *The same property applies to any similar figures, as well as to squares, described on the three sides of a right-angled triangle.*

Thus, a regular hexagon described on the hypothenuse is equal to the sum of two hexagons described on the base and perpendicular: or, a circle described on the hypothenuse as a diameter, is equal to the sum of the two circles described on the other sides.

Exercise 1. Make an equilateral triangle equal to the sum of two equilateral triangles, whose sides are 12 and 16, and say what is the length of the side.

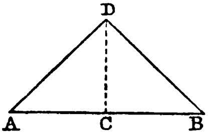
2. Make a hexagon equal to three times a hexagon whose side is ½ inch.

3. There are four circles whose diameters are 1, 2, 3, and 4: make a circle equal to all the four.

PROPOSITION LXVII. PROB.

To make a Square equal to half a given Square.

Let AB be a side of the given square: bisect it in C, by the perpendicular CD. Make CD equal to AC or CB. Draw AD, DB. Either of these will be the side of the required square.



Corollary. *The same method may be applied to any other figure as well as to a square.*

Thus, let it be required to construct a circle equal to half a given circle, whose diameter is AB. Bisect AB, and proceed as before; AD or DB will be the diameter of the required circle.

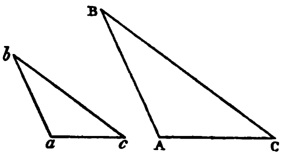
Exercise 1. Make a square equal to the half of a given square whose side is 2 inches.

2. Describe a circle equal to half a given circle whose diameter is 1½ inch.

3. There is an equilateral triangle whose side is 4 inches; construct an equilateral triangle equal to the fourth part of it.

PROPOSITION LXVIII. THEOREM.

If two Triangles (ABC and *abc*) *have the Angles of the one respectively equal to the Angles of the other, the Triangles are similar, and the Sides about the equal Angles are proportional.*

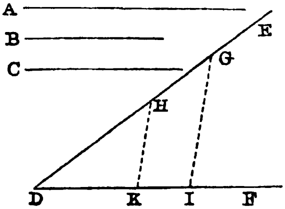


That is, AB : *ab* :: AC : *ac* :: BC : *bc*.

PROPOSITION LXIX. PROB.

To find a Fourth Proportional to three given Lines A, B, *and* C.

From any point D draw any two lines DE, DF. On these set off DG = A; DH = B; and DI = C. Join GI; and draw HK parallel to it. DK will be the fourth proportional to A, B, and C: that is, A : B :: C : DK.



COROLLARY. In the same manner we *find a third proportional to two given lines*, by merely using the second twice, that is, both for B and C.

EXERCISE 1. Three lines measure respectively 9, 12, and 15 feet: find a fourth proportional to them: state its length as measured on the scale, and prove it by arithmetic.

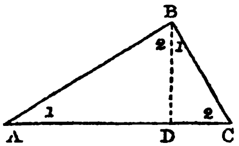
2. The same with the numbers 3, 4, and 5.

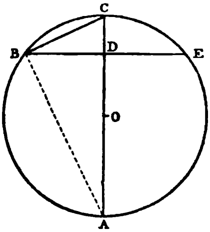
3. Find a third proportional (by Geometry) to the numbers 20 and 30.

PROPOSITION LXX. THEOREM.

In any right-angled Triangle (ABC), *if a Perpendicular* (BD), *be drawn to the Hypothenuse from the opposite angular Point, the Perpendicular will be a Mean Proportional between the Segments of the Base* (AD and DC). *And each Side will be a Mean Proportional between the adjacent Segment of the Hypothenuse, and the whole Hypothenuse.*

COR. 1. $BD^2 = AD \times DC$;
$AB^2 = AD \times AC$;
and $BC^2 = CD \times AC$.





Cor. 2. If BCE be an arc of a circle; BE its chord; AC a diameter perpendicular to the chord; BC the chord of half the arc BCE; and CD the height of the arc; then $BD^2 = AD \times DC$; and $BC^2 = AC \times CD$.

Proposition LXXI. Prob.

To find a Straight Line which shall be a Mean Proportional between two given Straight Lines AB and BC.

(See the diagram of the following Proposition).

Produce AB till BE be equal to BC. On AE describe the semicircle AFE. Draw BF perpendicular to AE. BF will be the line required.

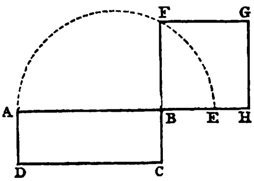
Exercise 1. The lengths of two lines are 4 and 9: find a mean proportional between them, and prove it by arithmetic.

2. Find, geometrically, a mean proportional between the numbers 108 and 243.

Proposition LXXII. Prob.

To construct a Square that shall be equal to a given Rectangle ABCD.

Proceed as in the last problem, and then on BF describe the square BFGH. That square will be equal to the given rectangle.

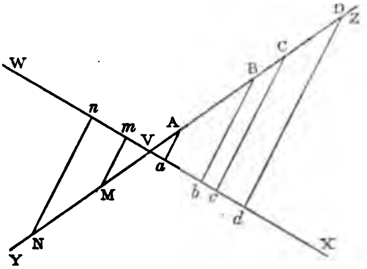


Exercise. The sides of a rectangle are 25 and 16: make a square equal to it, and state the length of the side.

PROPOSITION LXXIII. THEOREM.

If two unlimited diverging Straight Lines (WX and YZ) *are cut by any Number of Parallel Straight Lines* (A*a*, B*b*, M*m*, N*n*, &c.), *they are both cut in the same Proportion in all their Parts.*

Thus,—
AB : BC :: *ab* : *bc*,
VA : AB :: V*a* : *ab*,
VA : VD :: V*a* : V*d*,
MN : BC :: *mn* : *bc*,
&c. &c.

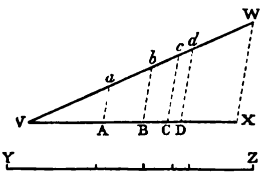


NOTE. Since the diverging lines are unlimited, we must not include among their parts, the remainders beyond the last of the parallel lines, towards W, X, Y, and Z.

PROPOSITION LXXIV. PROB.

To divide a given Straight Line (VX) *in the same Proportion in which another given Straight Line* (YZ) *is divided.*

Through V draw VW equal to YZ; and set off on VW the parts V*a*, *ab*, *bc*, &c. (or rather V*a*, V*b*, V*c*, &c.), equal to the several parts of YZ. Join WX, and parallel to it draw *a*A, *b*B, *c*C, &c. VX will then be divided as required, in the points A, B, C, &c.



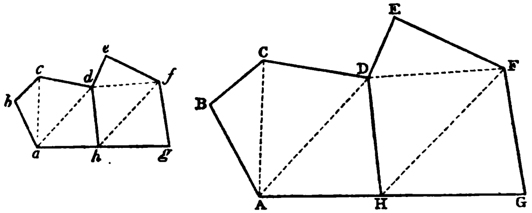
EXERCISE. The several parts of a line are 3, 5, 1, 4, making the whole line 13. The length of another line is 20: divide it in the same proportion as the first.

PROPOSITION LXXVI. PROB.

To construct a Figure similar but unequal to a given Plane Figure.

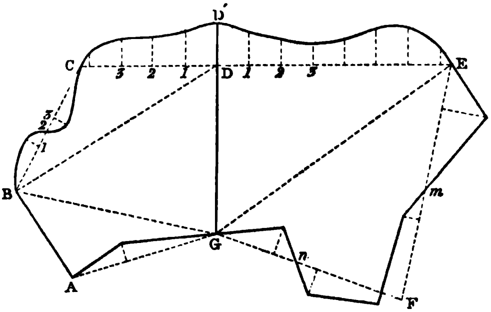
CASE 1.

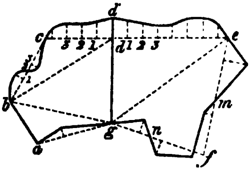
When the required Figure is to be drawn to a given Scale (No. 2), *the given Figure being on another given Scale* (No. 1).



If the boundary is not very irregular, that is, if it consist of a few straight lines, as ABCDEFGH, divide the given figure into triangles (as directed in Prop. VII, and Note 1). Commence with one of the largest of the triangles, as ADH, and measure its sides on the scale No. 1: then make a triangle, *adh*, with its sides equal to the same three numbers taken from the scale No. 2. Measure the sides AC, CD, of the next triangle on No. 1, and to *ad* apply a triangle *acd*, having its two other sides, *ac*, *cd*, equal to the measured lengths of AC, CD, but taken from the other scale, No. 2. Proceed thus till all the triangles are laid down.

If the outline, or any part of it, is very irregular, but still made up of straight lines, as from E to A in the following diagram, draw one or more straight lines as near it as may be convenient, as EF, FG, GA, and call this the straightened boundary. Then from the several angular points of the crooked boundary, drop perpendiculars on the straightened boundary. If any part of the boundary is curved, as from B to E, it may be treated in nearly the same manner, by drawing the perpendiculars from the most abrupt points of the curve, as in the portion BC; or, if the curvature is pretty equal, that is, without sudden turns, by drawing the perpendiculars at equal distances measured on the straightened boundary. When this is done, divide the space within the straightened boundary into triangles, measure their sides on the scale No. 1, and lay them down from No. 2, as before. Then on No. 1 measure the distances of the several perpendiculars and points of crossing, from the angular points of the straightened boundary, as B 1, B 2, B 3, D 1, D 2, &c., and set off the same on the straightened boundary of the new figure, as *b* 1, *b* 2, *b* 3, &c., from No. 2. Through the points thus marked draw perpendiculars to the straightened boundary, and measure the lengths of the perpendiculars on the given figure from No. 1, and when remeasured from No. 2, transfer them to the new figure. When that is finished connect the points marked on the perpendiculars with a ruler where the boundary consists of straight portions, but with the hand where it is curved, directing the hand according to the appearance of the curve in the given figure. This being done, the required figure is completed.





Note 1. All the lines (dotted on the diagram) which did not originally belong to the given figure, should, of course, be drawn only with the pencil, and rubbed out when the work is finished.

Note 2. The perpendiculars round the curved and crooked parts of the boundary are called *Offsets*.

Note 3. It is by precisely the same method that a field, farm, or estate is surveyed, that is, measured with the chain and laid down on a plan; the field, farm, or estate being the given figure, which we divide into triangles, straightening its boundaries and measuring its offsets exactly as we should do if it were on paper.

Exercise. Take the two preceding diagrams; measure the dimensions of the larger figure in each from a scale of 50 parts to the inch, and lay it down from a scale of 30 parts to the inch.

Case 2.

When the required Figure is to be constructed on a given Straight Line, corresponding to a given Side or Diagonal of the given Figure.

Let it be required, for instance, on a given straight line *fg* (see the first diagram in this problem), to construct a figure similar to ABCDEFGH, *fg* corresponding to FG, and no scale being given.

FIRST METHOD.

If the figure is simple in its parts, that is, bounded by a few straight lines, divide it into triangles, as before. Then, on *fg* construct a triangle, *fhg*, similar to FHG (by Prop. xxii). On *fh* construct another triangle, *fdh*, similar to FDH: and so on, till the figure is completed.

SECOND METHOD.

If the figure is more complicated (as in the second diagram of this Problem), take any scale for that of the given figure, and construct a scale for the required figure, bearing the same proportion to the former that the given side or diagonal of the required figure bears to the corresponding side or diagonal of the given figure. Then proceed as in Method 1st, Case i.

The scale for the required figure is to be constructed thus:—Take the given straight line for the second term of a proportion, and the corresponding side or diagonal of the given figure for the first term. For the third term, take any definite length (as 1000) from the scale assumed for the given figure. Then (by Prop. LXIX), find a fourth proportional to these three lines: this will be the length of 1000 equal parts on the required scale. That scale must then be divided in the usual way (by Prop. XVI and XVII).

PROPOSITION LXXXII. THEOREM.

In any two similar Figures, plane or solid, all the Sides, Diagonals, Boundaries, or other Lines of the one, are in the same proportion to the corresponding Sides, Diagonals, Boundaries, or other Lines of the other.

Thus, the diagonals of two squares are to each other as their sides: the circumferences of two circles are as their diameters: the arcs of two similar segments of circles are as their chords: the circumferences of two similar ellipses are as their corresponding axes, &c. For instance, if we double the diameter of a circle, we necessarily double the circumference also: if we treble the diameter, we treble the circumference; and so forth.

PROPOSITION LXXXIII. THEOREM.

The Areas of any two similar Surfaces are to each other as the Squares of their Sides, Diameters, or other corresponding Lines.

Thus, two equilateral triangles are to each other as the squares of their sides: the areas of two circles, or the surfaces of two spheres, are as the squares of their diameters, or of their circumferences: the areas of two similar segments are as the squares of their chords, &c. If, for instance, we double the diameter of a circle, we quadruple the area: if we treble the diameter, we make the area ninefold; and so on.

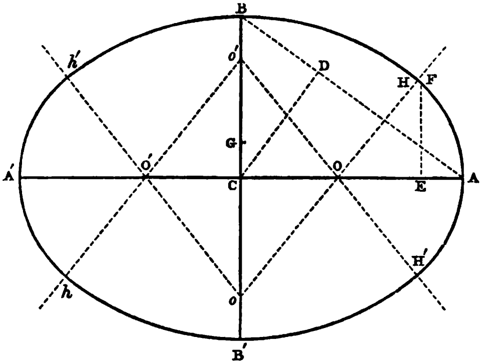
PROPOSITION LXXXIV. THEOREM.

Any two similar Solids are to each other as the Cubes of their Sides, Diameters, or other corresponding Lines.

Thus, two spheres are to each other as the cubes of their diameters or circumferences; two similar cones are as the cubes of their altitudes, &c. If, for instance, the diameter of one sphere be double that of another, its solid content is 8 times that of the other: if the diameter be treble, the solid content is 27 times. If a gallon and a bushel measure are made similar, the diameter and depth of the bushel will only be double those of the gallon, although a bushel holds eight gallons.

PROPOSITION LXXXVIII. PROB.

On two given Axes (AA′ *and* BB′) *to describe with the Compasses, and with four Centres, a Figure resembling an Ellipse, and agreeing with a true Ellipse in eight principal Points in the Circumference.*



Draw AB, and then CD perpendicular to it. Set off CE equal to AD, and draw EF perpendicular to AC, and equal to BD. Suppose AF to be drawn, and bisect it by a perpendicular, cutting AC in O. Set off BG equal to AO, and bisect OG by a perpendicular cutting B′C in *o*. Make CO′ equal to CO, and C*o*′ to C*o*. Draw *o*O, *o*O′, *o*′O, and *o*′O′, and produce them. With the centres O and O′, and radius OA, describe the two arcs HH′ and *hh*′, the former of which will pass through F, both being terminated by the produced lines *o*O, &c. With the centres *o* and *o*′ and radius *o*B describe the arcs H*h*′ and H′*h*, which, if the work is nicely done, will exactly meet the two arcs previously described. The figure is now completed.

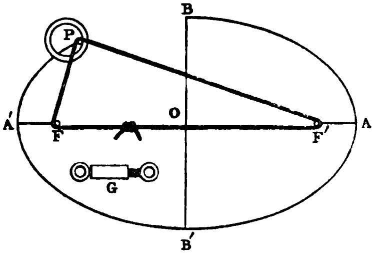
Note. The outline described touches the circumference of a true ellipse in the points A, B, A′, B′, and cuts it in the point F, and in the three points corresponding to F in the other quadrants. When the axes are not very unequal, the figure will not deviate much, either in appearance or in measurement, from a true ellipse.

There are several lines which do not appear in the diagram, since nothing but their extremities or intersections with other lines are wanted. In fact, after F has been found, O may be found by trials; and, when O and G have been found, *o* may be found by trials.

Proposition XC. Prob.

On two given Axes, AA′ *and* BB′, *to construct a true Ellipse.*

With the centre B, and radius = AO, cut AA′ in F and F′. In these points (which are the foci of the ellipse), stick two pins. These must either be pressed firm into a drawing-board beneath the paper, or held fast by an assistant. Take a string, double it, and tie its ends together in such a manner that, when doubled, it shall reach from F to A. Then, with the point of a pencil or drawing-pen, P, stretch the string, and sweep the pen or pencil slowly round: it will describe the ellipse required.



Note. In doing this, observe particularly that the pen or pencil-point exactly reach the points A, B, &c.; and, if not, the string must be tied shorter or longer till it do so. To keep the string from slipping below the pen or pencil-point, place the latter within a small ring between the thread and the paper, carrying the ring round along with the pen or pencil.

The string should be of some substance very flexible, and as little elastic as possible. A thread of unspun silk, or a strong fibre of flax, answers very well: a hair, from its elasticity, is very unsuitable for the purpose.

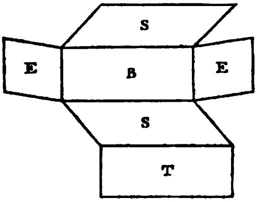
If a small adjusting screw, G, be used for connecting the two ends of the string, and regulating its exact length after it is previously tied to the two loops at the ends of the screw, as near the true length as possible, it will be found very convenient.

Exercise 1. Describe an ellipse, having its axes 3 and 2 inches respectively.

2. On the same axes describe a resemblance of an ellipse (by Prop. lxxxviii), and observe how far they differ.

Proposition XCIX. Prob.

To construct, with Pasteboard, a Parallelopiped, whose Sides are given Parallelograms.



Take any two opposite sides of the parallelopiped for the top and bottom; any other two opposite each other for the ends; and the remaining two for the sides peculiarly so called.

Draw a parallelogram B, having its sides and angles equal to those of the bottom of the proposed parallelopiped. Above this on its upper side, and below it on its lower side, construct a couple of parallelograms, S and S, having their sides and angles equal to those of the two sides of the parallelopiped, but the oblique lines not parallel in the two parallelograms, but inclined outward towards the same end of the parallelopiped. In the same manner construct the two ends E and E. Upon the lowest side of the lowest parallelogram before drawn, construct another parallelogram, T, having its sides and angles equal to those of B, but its ends, if oblique, inclined opposite ways.

This being done, cut the outline of the figure through the pasteboard, and the other lines half through. The sides and ends being then folded towards the opposite side of the pasteboard, till they meet, will form the proposed parallelopiped.

Exercise 1. Construct a one-inch cube.

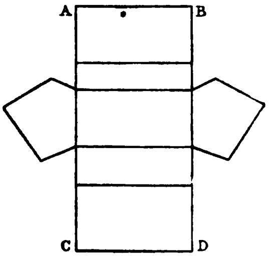
2. Construct a rectangular parallelopiped having its top and bottom each 1 inch by $\frac{1}{2}$, and its ends $\frac{1}{2}$ inch by $\frac{1}{4}$.

3. Construct an oblique-angled parallelopiped having its top and bottom each a rectangle 1½ inch by ¾, and its rising edges each ½ inch, the acute angles of the parallelograms forming the sides and ends being all of 60°.

PROPOSITION C. PROB.

To construct, with Pasteboard, a right rectilineal Prism of given Dimensions.

Draw a line, AB, equal to the length of the prism, and BD and AC perpendicular to it. On each of these set off lengths equal to the widths of the sides of the prism taken in order, and connect their extremities by parallel straight lines completing a series of rectangles. On the two opposite ends of any one of these rectangles, describe two figures equal and similar to the ends of the figure, and turned opposite ways. Cut the figure and fold it as in the last problem. The edges which meet may then be sewed or pasted together.



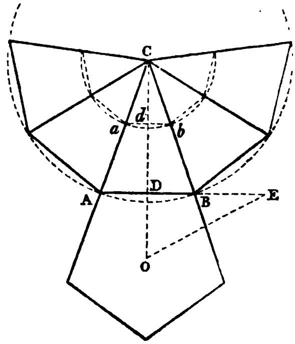
EXERCISE. Construct a regular hexagonal prism.

NOTE. A *cylinder* may also be constructed in the same manner, the ends being circles of the given diameter, and the rectangle ABDC having its length equal to their circumference, which may be found by taking 3⅐ times the diameter, or simply by wrapping the pasteboard round the circular end.

PROPOSITION CI. PROB.

To construct a regular right Pyramid of given Dimensions, with Pasteboard.

Construct the base of the pyramid O. On one side of this construct a triangle, ABC, equal to one of the triangles forming the sides of the pyramid. With the centre C and radius CA or CB describe a circle. With the distance AB in the compasses, step along the circumference as many times (including AB) as there are sides to the base. Some of these steps may be on one side of AB and some on the other. Connect the successive steps with the centre, and with each other, by straight lines; and cut and fold the figure as in the two previous problems.



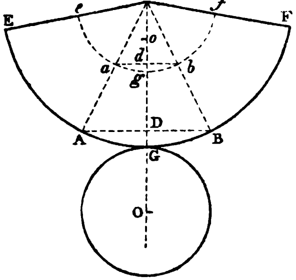
NOTE. To construct the triangle ACB, find the centre of the base, O: draw OC perpendicular to AB; and, if the slant height of the pyramid is given, make CD equal to it: but if the edge of the pyramid is given, make AC and BC each equal to it; or, if the perpendicular altitude of the pyramid is given, set off DE equal to it, and make CD = OE.

EXERCISE. Construct a hexagonal pyramid having its perpendicular altitude 2 inches, and each side of its base a half inch.

PROPOSITION CII. PROB.

To construct a right Cone of given Dimensions, with Pasteboard.

Construct an isosceles triangle, ACB, having its base AB equal to the diameter of the cone's base, and its altitude CD equal to the perpendicular height of the cone, or its sides AC and BC each equal to the slant height of the cone. Measure the distances CB and AB on any scale, and find by arithmetic a fourth proportional to CB, AB, and 90°. Make the angles DCE and DCF each of the number of degrees in the fourth proportional just found.



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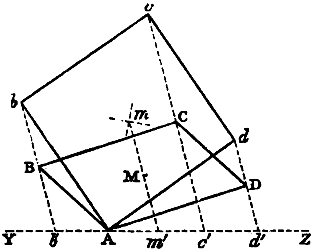
With the centre C and radius CA describe the arc EGF. On CD produced, set off OG equal AD. With the centre O and radius OG describe a circle for the base of the cone, and cut out the figure consisting of the sector and circle, and fold it as before.

Exercise. Construct a cone having the diameter of its base 1 inch, and its slant height the same.

Proposition CIV. Prob.

To draw any given horizontal rectilineal Figure in Geometrical Perspective.[*]

Having drawn the given figure A*bcd* of its true dimensions, taken from any scale, and in its true position on the paper (which, at this stage, is regarded as a horizontal plane), through the nearest angular point, A, draw a straight line, YZ, across the paper. The paper will now be supposed to be placed vertically, and YZ, being still horizontal, is called the *Ground Line*. From all the angular points draw straight lines, *bb*′, *cc*′, &c., parallel to each other, and making with the ground line any angle which may be thought most suitable. Bisect each of these parallel lines, or divide them in any other proportion which may be preferred, in the points B, C, &c. (all, however, in the same proportion), and join these points by the straight lines AB, BC, &c. ABCD will be the figure required.



Cor. 1. Any *point* within the figure, as *m*, may have its perspective place M assigned in the same manner.

Exercise. The scholar may construct a regular pentagon in different positions, and draw it in perspective.

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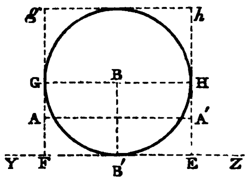
[*] By the term "geometrical perspective," I mean that kind of perspective in which the eye of the spectator is supposed to be at an infinite distance. For the sake of simplicity, the paper is always supposed to be vertical, and the base of any solid figure usually horizontal.

PROPOSITION CV. PROB.

To draw a given Circle in Geometrical Perspective.

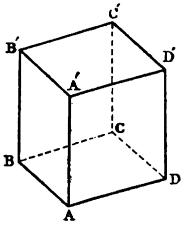
Describe a square, EF*gh*, about the circle, and having one of its sides in the ground line YZ. Bisect the two sides perpendicular to this (or divide them in the proportion chosen) in the points G and H. EFGH will be a perspective representation of the square. Draw the straight lines AA′ and BB′ bisecting the opposite sides of the rectangle EFGH. AA′ and BB′ will be the axes of the required ellipse. On these describe the ellipse by Pr. xc, or approximately by Pr. lxxxviii.



PROPOSITION CVI. PROB.

To draw, in Geometrical Perspective, any solid rectilineal Figure having its Base horizontal and its Sides vertical.

Draw its base in perspective, as ABCD (by Prop. civ), and from all its angular points draw lines perpendicular to the ground line. On these set off AA′, BB′, &c., equal to the lengths of the upright edges of the figure, measured from the same scale as the base, and draw the lines A′B′, B′C′, &c., and the solid figure is completed.



NOTE. When the drawing is finished, those lines which are behind the solid, and concealed from view, should not appear; or, at least, if they appear at all, should be drawn dotted.

EXERCISE 1. Draw a cube in various positions.

2. Draw, in various positions, a rectangular parallelopiped having its base 2 inches by 1, and its height 3.

PROPOSITION CVII. PROB.

To draw a right Cylinder in Perspective.

CASE 1.

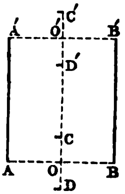
When the Ends are horizontal.

FIRST METHOD.

Draw the circular base in perspective (by Prop. cv), and take any number of points in the circumference of the ellipse, and proceed with these points as with the angular points of the base in the last proposition, drawing vertical lines from all of them of an equal length: the summits of these vertical lines will be a series of consecutive points, which, when connected by the hand, will form another ellipse for the top of the cylinder.

SECOND METHOD.

Having put the base into perspective (by Prob. cv), let the axes of the elliptic base thus drawn be AB and CD, and its centre O. Draw the vertical line OO′ equal to the length of the cylinder. Through O′ draw A′B′ perpendicular to OO′; and set off O′A′, O′B′, O′C′, O′D′, respectively, equal to OA, OB, OC, and OD. On A′B′ and C′D′, as axes, describe another ellipse for the top of the cylinder, and draw AA′ and BB′ for its sides.



CASE 2.

When the Ends of the Cylinder are vertical.

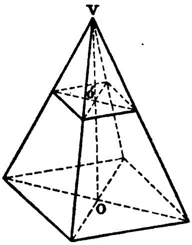
Having drawn the circular end of its true size and position, draw a tangent to it along the paper instead of across it: use this tangent as a ground line; and then proceed as in Case 1.

EXERCISE. Draw a cylinder—length 2 inches, diameter 1.

PROPOSITION CVIII. PROB.

To draw, in Geometrical Perspective, a regular right Pyramid, or a right Cone, having its Base horizontal.

Having drawn the base of the true figure and dimensions, find its centre. Draw the base and the centre in perspective (by Prop. CIV or CV). From the perspective centre, O, draw a vertical line, OV, equal to the perpendicular height of the pyramid or cone; and, from V, in the case of the pyramid, draw lines to all the angular points of the base, or, in the case of the cone, draw two straight lines touching the elliptic base, and the figure is completed.



COR. *For a frustum of a pyramid.*—Having drawn the whole pyramid as above, on OV set off OO′ equal to the perpendicular height of the frustum. From O draw straight lines to the angular points of the base, and from the point O′ draw lines parallel to these, until they meet the same slant edges of the pyramid, and connect the points where they meet them by straight lines, which will be parallel to the sides of the base, and will form the top of the frustum.

For a frustum of a cone.—Having drawn the whole cone as above, draw straight lines from V to the extremities of the axes of the base: set off OO′ = the perpendicular height of the frustum; and through O′ draw lines parallel to the axes of the base, and terminated by the lines last drawn from V: these will be the axes of the ellipse which is to form the top of the frustum.

EXERCISE. Draw, in two positions, a pentagonal pyramid, having its altitude 2 inches, and each side of the base $\frac{1}{2}$ inch. Then cut off a frustum of the same pyramid of half its height.

CHAPTER III.

MENSURATION OF LINES.

TABLE OF LINEAL MEASURES, ETC.

12 inches = 1 foot
3 feet = 1 yard
$5\frac{1}{2}$ yards = 1 pole*
40 poles = 1 furlong
8 furlongs = 1 mile
3 miles = 1 league

5280 feet = 1760 yards = 80 chains = 1 mile.

22 yards = 4 poles = 1 chain of 100 links.

7·92 inches = 1 link.

$\pi = 3{\cdot}1416 - .$

$\frac{\pi}{4} = 0{\cdot}7854 - .$

$\frac{\pi}{6} = 0{\cdot}5236 - .$

$\frac{1}{\pi} = 0{\cdot}31831 - .$

$\frac{1}{4\pi} = 0{\cdot}07958 - .$

$\pi^2 = 9{\cdot}8696 + .$

Problem I.

Any Two Sides of a Right-angled Triangle being given, to find the Third Side.

Rule 1. *When the legs are given, to find the hypothenuse.* Add together the squares of the two legs, and extract the square root of the sum.

Rule 2. *When the hypothenuse is given and one of the legs, to find the other leg.* Subtract the square of the given leg from the square of the hypothenuse, and extract the square root of the difference.

FORMULÆ.

$$h = \sqrt{b^2 + p^2}, \qquad b = \sqrt{h^2 - p^2}, \qquad p = \sqrt{h^2 - b^2}.$$

h being the hypothenuse; b, the base; and p, the perpendicular.

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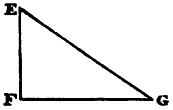
* Called also *a rod*, and *a perch*.

COR. Hence the diagonal of a square is equal to the square root of twice the square of the side. And the side of a square is equal to the square root of half the square of the diagonal.

EXAMPLE 1. The two sides of a rectangular field measure respectively 5·48 and 3·86 chains: what is the length of the diagonal?

Here the one half of the rectangle is a right-angled triangle, of which we have $b=5{\cdot}48$, and $p=3{\cdot}86$.

$$\begin{array}{rr}
b=5{\cdot}48 & 3{\cdot}86=p \\
5{\cdot}48 & 3{\cdot}86 \\
\hline
43\,84 & 23\,16 \\
219\,2 & 308\,8 \\
2740 & 1158 \\
\hline
30{\cdot}03\,04 & 14{\cdot}89\,96=p^2 \\
 & 30{\cdot}03\,04=b^2 \\
\cline{2-2}
\end{array}$$



$$\begin{array}{l}
h^2=b^2+p^2=44{\cdot}93\,00\,(6{\cdot}70+\;=h \\
36 \\
\hline
127)893 \\
889 \\
\hline
1340)400
\end{array}$$

Ans. 6·70 chains.

EXAMPLE 2. The base of a right-angled triangle is 4 ft. 6 in., and the hypothenuse 7 ft. 5 in.: what is the perpendicular?

$$\begin{array}{rrr}
b=54\text{ in.} & h=89\text{ in.} & \\
54 & 89 & \\
\hline
216 & 801 & 50,05(70{\cdot}7+ \\
270 & 712 & 49\phantom{,05(70{\cdot}7+} \\
\hline
b^2=2916 & h^2=7921 & 1407)10500 \\
 & & 9849 \\
 & b^2=2916 & 651 \\
\hline
 & p^2=5005 & 12)70{\cdot}7+\text{in.}=p.
\end{array}$$

5 ft. 10·7+in. Ans.

EXERCISE 1. The base of a right-angled triangle is 513, and the perpendicular 684 feet: what is the hypothenuse?

Ans. 855 feet.

2. The hypothenuse of a right-angled triangle is 2 ft. 10 in., and the base 2 ft. 6 in.: what is the perpendicular?

Ans. 1 ft. 4 in.

3. The side of a square is 9·774 feet: what is the length of the diagonal?

Ans. 13·823 – feet.

4. What is the side of a square whose diagonal is 8 ft. 5 in.?

Ans. 5 ft. $11\frac{1}{2}$ – in.

5. A ladder is to be placed so as to reach a window, the sill of which is $67\frac{1}{2}$ feet from the ground: the foot of the ladder cannot be brought nearer than 36 feet from the wall: what length of ladder will be sufficient?

Ans. $76\frac{1}{2}$ feet.

6. The diagonal of a rectangular field measures 10·71 chains, and the one side 9·45: of what length is the other?

Ans. 5·04 chains.

7. The two sides of an isosceles triangle measure, each, 65 yards, and the base 50 yards: what is the altitude?

Ans. 60 yards.

8. The chord of an arc is 10 and its height 4: the chord of half the arc is required.*

Ans. 6·403 +.

9. The chord of an arc is $16\frac{1}{2}$, and the chord of half the arc $11\frac{3}{10}$: what is the height of the arc?

Ans. 7·72 +.

10. The vertical axle of a thrashing-mill, driven by horses, is placed with its centre $10\frac{1}{2}$ feet from a wall, but each shaft or arm attached to the axle is 15 feet in length, measured from the centre of the axle: how much of the wall must be taken down to allow it to revolve, no additional room being required for the horse, since it does not reach beyond the end of the shaft?

Ans. 21·4 + feet.

11. A ladder, 50 feet long, being placed in a street, reached a window 48 feet from the ground, on one side of the street; and by turning it over without removing the foot, it reached another window 40 feet high on the other side: the breadth of the street is required.

Ans. 44 feet.

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* In such questions the learner should have a diagram before him, and if this is drawn correctly, from a scale, the geometrical operation will prove the arithmetical.

12. A ladder, standing upright beside a wall 100 feet high, just reaches the top: how far may the foot of the ladder be removed from the wall, and still reach within 6 inches of the top? Ans. 10 − feet.

13. Each side of the base of a square pyramid measures 100 feet, and its perpendicular altitude also 100: what is the length of the slant side and of the slant edge?[*]

Ans. $111\frac{4}{5}+$ and $122\frac{1}{2}-$.

14. What is the altitude of an equilateral triangle whose side is 1? Ans. ·866025 +.

PROBLEM VI.

To find the Circumference of a Circle, when the Diameter is given.

RULE. Multiply the diameter by 3·1416 −.

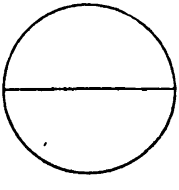
FORMULA.

$$C = D \times \Pi\,[\dagger]$$

REV. FORMULA.

$$D = C \div \Pi.$$

$$\text{or, } D = C \times \frac{1}{\Pi}.$$



D being the diameter; C, the circumference; and Π the number 3·141,592,65 +.

EXAMPLE 1. The diameter of a circle is 39·7: what is the circumference correct to five figures?

```
   3·1416‡
     39·7
  ———————
   219912
  282744
 94248
 ————————
124·72152        Ans. 124·72 +.
```

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[*] For questions like this, the scholar should have models of the solid figures. For this question he would require a pyramid cut in two through the vertex

[†] Π, the Greek P (pronounced in England like the English word *Pie*), is always used for the number representing the circumference of a circle whose diameter is 1, that is, the number 3·1416 —,

$\frac{1}{\Pi}$ is the reciprocal of Π, and is ·31831 −. (See Table of Lineal Measures).

[‡] 3.1416 is so near to 3·14159+ that it may be used with safety to obtain results true to five figures.

Example 2. Required, the equatorial diameter of the earth, the circumference being 24896. Let the answer be correct to four figures.

$$\begin{array}{r l}
3{\cdot}1416)24896{\cdot}0000(7925-^{*}, & \text{Ans.}\\
21991\ 2 & \\
\hline
290480 & \\
282744 & \\
\hline
77360 & \\
62832 & \\
\hline
145280 & \\
157080 & \\
\hline
-11800 &
\end{array}$$

Exercise 1. What is the circumference of a circle, the diameter being 758? Ans. 2381·3 + .

2. The diameter being 18$\frac{3}{7}$, what is the circumference? Ans. 57$\frac{9}{10}$, nearly.

3. The radius is 2$\frac{7}{8}$: find the circumference. Ans. 18·064 + .

4. Compute the semicircumference when the radius is ·3183. Ans. 1.

5. What is the circumference of a coach wheel of 2 feet 5$\frac{1}{2}$ inches radius? Ans. 15 feet 5$\frac{3}{8}$ in. nearly.

6. Calculate the diameter of a circle, the circumference of which is 6.224. Ans. 1·981 + .

7. What must be the diameter of a wheel to turn round 1000 times in travelling a mile? Ans. 1 ft. 8·17 – in.

PROBLEM VII.

Having given the Chord (BE) *and the Height* (CD) *of an Arc* (BCE) *of a Circle, to find its Radius or Diameter.*

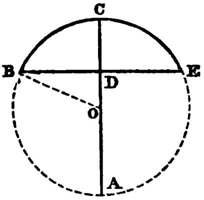
Rule. Divide the square of half the chord, by the height, the quotient will be (AD) the remainder of the diameter (AC): to this add the height (CD): the sum will be the diameter, and half of that the radius.

---

* The last figure in the quotient is made 5, not 4, because 5, though too much, is nearer the truth than 4. In this case the sign — is placed after the 5 in the quotient, and before the remainder, which is found, in this case, by subtracting the upper line from the lower.

FORMULÆ.

$$AD = BD^2 \div CD.$$
$$AC = AD + CD.$$



Note. The two *Reverse* cases of the problem are resolved by means of the right-angled triangle BDO.

Example. The chord of an arc was found by measurement to be 8, and the height of the arc 3: what was the radius of the circle to which the arc belonged?

$$BD^2 = 4^2 = 16 \qquad 3)\underline{16}$$
$$5\tfrac{1}{3} = AD$$
$$\underline{3 \;\; = CD}$$
$$2)\underline{8\tfrac{1}{3} = AC}$$
$$\text{Ans.} \qquad 4\tfrac{1}{6} = OC.$$

Exercise 1. The height of an arc is 1·5, and the chord 10·8: determine the radius with which the arc was described. Ans. 10·47.

2. What was the radius of an arc of which the chord was equal to the height, each being $5\frac{1}{4}$ feet?
Ans. 3·28125, the segment being greater than a semicircle.

3. The chord of an arc is 54, and the radius 100: what is the height of the arc? Ans. 4.

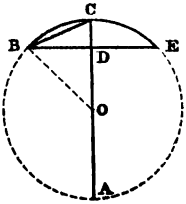
4. The height of an arc is ·7, and the radius 1: what is the chord of the arc? Ans. ·3918 + .

Problem VIII.

Having given the Height (CD) *of an Arc of a Circle* (BCE), *and the Chord of half the Arc* (BC), *to find its Radius or its Diameter.*

Rule. Divide the square of the chord of half the arc by the height of the arc. The quotient will be the diameter.

FORMULA.	REV. FORMULÆ.
$\text{Diam} = \frac{BC^2}{CD}$.	$CD = BC^2 \div \text{Diam}$. $BC = \sqrt{(\text{Diam} \times CD)}$.



NOTE. The *Reverse* cases of this problem may also be resolved by means of right-angled triangles.

EXERCISE 1. The height of an arc was found by measurement to be 7 ft. 9½ in.; and the chord of half the arc 15 ft. 7 in.: with what radius had the arc been described? Ans. 15 ft. 7 in.

2. The radius of a circle is 4½; and the versed sine of an arc of the same circle, 4: what is the chord of half the arc? Ans. 6.

PROBLEM IX.

To find the Length of an Arc of a Circle, when the Number of Degrees it contains, and the Circumference, are given.

RULE. Say, As 360°, to the number of degrees in the arc, so is the whole circumference, to the length of the arc.

FORMULA. $\text{Arc} = \frac{C \times N}{360}$.

C being the circumf. and N the number of degrees.

EXAMPLE. Required the length of an arc of 107½ degrees, the whole circumference being 156.

As 360° : 107½° :: 156
107½
78
1092
156
1,0)1677,0
4)1677
9)419·25
Ans. $46{\cdot}58\dot{3}$

161

Exercise 1. An arc contains 57° 38′, the circumference of the circle to which it belongs being 68 yards: compute the length of the arc. Ans. 10 yds. 2·66 – ft.

2. The Equatorial Circumference of the Earth is about 24,896 miles: what is the length of a degree of longitude at the Equator? Ans. $69{\cdot}1\dot{5}$ miles,

PROBLEM X.

To find the Length of an Arc of a Circle, having given the Number of Degrees which the Arc contains, and the Radius or the Diameter of the Circle.

Rule. Find the circumference (by Pr. vi), and then the length of the arc (by Pr. ix).

$$\text{Formula.}\quad \text{Arc} = \frac{D \times \Pi \times N}{360}.$$

D being the diameter, and N the number of degrees.

Exercise 1. What is the length of an arc of 36° 42′ 16″, the diameter of the circle being 8, by Rule 1?

Ans. 2·5625 – .

2. What is the length of an arc of 198° $7\frac{3}{10}'$, the radius of the circle being $12\frac{2}{3}$, by Rule 1? Ans. 43·800 – .

PROBLEM XII.

To find (by the Tables in this Volume) the Length of an Arc of a Circle, having given the Radius of the Circle, and the Sine or Versed Sine of the Arc.

Rule. Divide the sine or versed sine by the radius, and look for the quotient in Table ii, in the column headed "N. Sine," or "N. V. Sine." Take the number standing opposite this, in the column headed "Arc," and that number, multiplied by the radius, will be the answer.

Note 1. If the arc is greater than a quadrant, find its supplement and subtract it from the semicircumference; or, more concisely, after finding, by the rule, the supplementary arc in the table, subtract it from 3·1416, and multiply the remainder by the radius.

NOTE 2. If the *chord* is given, we may take half the chord as a sine, compute its arc by the Rule, and double the arc thus found, for the answer.

If the *height* of the arc is given, take it as a versed sine, compute its arc by the Rule, and double the arc thus found, for the answer.

EXAMPLE 1. What is the length of an arc the sine of which is 7, the diameter of the circle being 16?

$$\text{Rad}=8)\underline{7{\cdot}0}$$
$$\cdot 8750.$$

Looking for ·8750 in the column headed "N. Sine," the nearest number to be found is ·8746; opposite which is 1·0647. This, multiplied by 8, gives 8·5176, of which we can take only the first three figures as certain.

∴ 8·52 – is our Answer.

EXAMPLE 2. The chord of an arc being 10 and the radius the same, the length of the arc is required.

Taking half the arc, as directed in Note 2,

$$\text{Rad}=10)\underline{5{\cdot}0}=\text{Sine}.$$
$$\cdot 5$$

Finding ·5 in the column "N. Sine," we have, opposite it, in the column "Arc," ·5236. This, multiplied by 10, is 5·236 for the half arc; which doubled is 10·47 +, for the Answer.

EXERCISE 1. The sine of an arc is 21, the radius being 24: what is the length of the arc? Ans. 25½ nearly.

2. Compute the length of an arc whose diameter is 20, and versed sine 5. Ans. 10·472.

3. The diameter of a circle being 1800, find the length of an arc of that circle whose chord shall be one-third of the diameter. Ans. 612 nearly.

4. Calculate the length of an arc of a circle whose radius is 725 and height 700. Ans. 222(7)*.

---

[*] The figures enclosed in parentheses are those in which the answer found by the rule deviates from the true answer.

5. The height of an arc is 50 and its chord 240 : determine, from the Tables, the length of the arc.

Ans. 266·+.

6. The chord of an arc is 40, and the chord of half the same arc is 25 : the arc is required. Ans. $53\frac{1}{2}$ nearly.

7. The sine of an arc greater than a quadrant, is 1·8, and the diameter of the circle 18 : what is the arc? (See Note 1). Ans. 26·5.

8. The chord of an arc is 120, and its height 144 : what is the length of the arc? (See Notes 1 and 2).

Ans. 398 –.

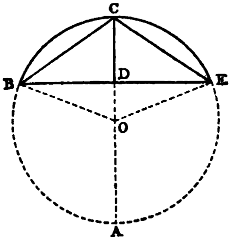
PROBLEM XIII.

To find the Length of an Arc (BCE) *approximately* (*without Tables*).

RULE. From eight times the chord of half the arc subtract the chord of the whole arc, and divide the remainder by 3.

FORMULA. $\text{Arc} = \dfrac{8c' - c}{3} + .$*

c being the chord of the whole arc; and c' that of half the arc.



NOTE. When the arc exceeds 90°, we find its supplement, and subtract it from the semicircumference. If it exceed 180°, we should find its excess above the semicircumference and add it; unless it is so large as to exceed 270°, when we should find what it wants of the whole circumference. If the arc is *about* 90, 180, or 270 degrees, we may proceed either way.

EXAMPLE. What is the length of the arc BCE, whose chord (BE) is 48, and radius 25?

First, in the triangle BDO, BD = 24, BO = 25, ∴ (by Problem I) DO = 7, and CD = CO – DO = 25 – 7 = 18.

---

* When the sign + or — is placed after a formula, it indicates that the formula is only approximate, + showing that the true answer is something more than that found by the formula, and — that it is something less.

Again, in the right-angled triangle BDC (by Prob. I) $BC = \sqrt{(BD^2 + CD^2)} = \sqrt{(24^2 + 18^2)} = 30$. Then

$$\begin{array}{r l} 30 & = BC \\ 8 & \\ \hline 240 & \\ 48 & = BE \\ \hline 3)192 & \\ \hline 64 & \text{ Ans.} \end{array}$$

Exercise 1. The chord of the whole arc is $50\frac{4}{5}$, and the chord of half the arc is $30\frac{3}{5}$: the length of the arc is required. Ans. $64(\frac{2}{3})$.

2. The length of the chord of the whole arc is 36·75, and the length of the chord of half the arc 23·2: what is the length of the arc? Ans. 50 –.

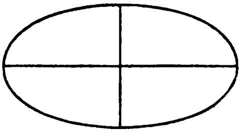
3. The span of the circular arch of a bridge is $48\frac{1}{2}$, and its rise $18\frac{1}{4}$: what is the length of the arch, measuring round the curve? Ans. 65 nearly.

4. If the chord of the whole arc be 16, and the radius of the circle 10, what will be the length of the arc? Ans. 18·5(18).

5. If the height of the arc is 4 and the diameter of the circle 30, what is the length of the arc. Ans. 22·4(13).

Problem XIX.

Having given the Axes of an Ellipse ($2a$ and $2b$), to find its Circumference.



Rule 1. Multiply half the sum of the two axes by 3·1416, and the product will be the circumference *exact enough for most practical purposes*, unless the ellipse is very eccentric.

Formula. $\text{Circumf.} = (a + b) \times \pi + .$

Rule 2. Multiply the square root of half the sum of the squares of the two axes by 3·1416, and the product will be the circumference *more nearly*.

Formula. $\text{Circumf.} = \sqrt{2(a^2 + b^2)} \times \pi - .$

165

Exercise. The axes of an ellipse being 3 and 2, required the circumference, by both rules.

Ans. 7·(85 +), and 8·(01 –).

Problem XXVI.

Having given two Lineal Dimensions in any Figure, Superficial or Solid, and, of two corresponding Dimensions in a similar Figure, having given one, to find the other.

Rule. State a proportion having, for its second term, the given dimension of the second figure, and the corresponding dimension of the first figure for the first term, the remaining given dimension being the third term. The fourth term, when found, will be the answer.

Formula. $$\begin{cases} A : a :: B : b, \\ \text{or } A : B :: a : b. \end{cases}$$

A and B being the two given dimensions in the first figure; and a and b, the corresponding dimensions in the second figure.

Example. If the side of a square be 3, and its diagonal 4·24, what will be the side of a square whose diagonal is 4?

As 4·24 : 4 :: 3 : 2·83, or
As 4·24 : 3 :: 4 : 2·83. Ans.

Exercise 1. A house is 60 feet long and 25 broad. In a ground plan of that house, the length, as laid down, is 5 inches: what ought to be the breadth?

Ans. $2\frac{1}{12}$ inches.

2. On a front elevation of the same house, drawn on the same scale, I find the height measures 3·75 inches: what was the actual height of the house? Ans. 45 feet.

3. If $11\frac{1}{2}$ inches be the length of the foot of a man 6 feet high, what was the length of Goliath's foot, supposing him to have been of the same proportion, and his height to have been, according to Bishop Cumberland's computation, 11 feet 10 inches? Ans. $22\frac{49}{72}$ inches.

4. I wish to ascertain, from a map of England, the distance, in a straight line, from Carlisle to Plymouth. No scale is attached to the map, but I take a given distance which I know, as that from Bristol to London, 108 miles. This, on the map, measures 3·8 inches, while the distance

from Carlisle to Plymouth measures 11·44 inches: what is the true distance of the two last mentioned places?

Ans. 310 + miles.

5. If the side of an equilateral triangle be 1000, and its altitude 866, what will be the altitude of an equilateral triangle whose side measures 11 feet 4 inches?

Ans. 9 ft. $9\frac{3}{4}$ + in.

6. When the diameter of a circle is 7, its circumference is 22 nearly: what will be the diameter of a circle whose circumference is 12? Ans. $3\frac{9}{11}$ nearly.

7. The side of an octagon being 1, the radius of the inscribed circle is 1·2071, and that of the circumscribed circle is 1·3066: what will be the diameter of the circle inscribed in another octagon, that of the circle described about it being 6? Ans. 5·543 + .

8. What length of paper will be required for a map of Great Britain, on the scale of $\frac{1}{40}$ of an inch to the mile; the greatest distance to be taken in, lengthways, being 600 miles, and a margin being allowed of $2\frac{1}{2}$ inches?

Ans. 20 in.

9. A tree 126 feet high, is 31 feet in circumference at its base: what will be its circumference 42 feet from the ground, supposing the trunk to be a perfect cone?

Ans. 20 ft. 8 in.

11. Wishing to know the height of a spire, whose summit was inaccessible, I found that the shadow of the said summit fell on a point on the same level with the base, and 132 feet distant from the centre of the base. Having previously erected an upright pole, 12 feet in height, I found its shadow, at the same time, to be 9·86 feet in length, measured on the horizontal surface on which it stood. What was the height of the spire?

Ans. $160\frac{1}{2}$ feet, nearly.

CHAPTER IV.

MENSURATION OF SURFACES.

A square whose side is 1 inch, 1 foot, 1 yard, &c., is called *a Square Inch*, a Square Foot, a Square Yard, &c.

The *Area* of a figure means the number of square inches, square feet, square yards, or other superficial denomination, which the figure contains, or is equal to.

Thus the area of the figure represented by the rectangle in the following diagram is 15 square inches, because it contains 15 of the small squares, each 1 inch long, and 1 inch broad.

TABLE OF SUPERFICIAL MEASURES.

144 square inches = 1 square foot.
9 square feet = 1 square yard.
$30\frac{1}{4}$ square yards = 1 square pole.
40 square poles = 1 rood.
4 roods = 1 acre.
640 acres = 1 square mile.
4840 sq. yds. = 160 sq. poles = 1 acre.
1210 sq. yds = 1 rood.
10,000 square links = 1 square chain.
10 sq. chains, or 100,000 sq. links = 1 acre.
16 square poles = 1 square chain.

GENERAL NOTE.

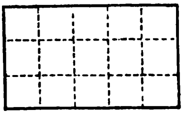
Throughout all the Problems in Part IV, the letter *A* is used to denote the *Area* of the figure to which the problem applies.

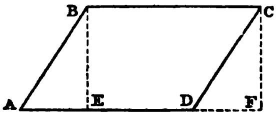
PROBLEM I.

To find the Area of a Square, Rectangle, or other Parallelogram, having given its Length and Breadth.

RULE. Multiply the length by the breadth.

NOTE 1. In the case of an oblique-angled parallelogram, when we speak of the breadth, we mean the perpendicular breadth, as BE in the second of the two following figures, not AB or DC.





FORMULA. Area $= L \times B$.

L being the length, and *B* the breadth.

In the case of the Square.

Area = Side^2. Side = $\sqrt{}$ Area.

NOTE 2. *Side*^{2} means the number, which expresses the side, multiplied by itself. Hence the product of any number by itself is called the "square" of that number.

NOTE 3. If the area of a square, whose side is to be found, is expressed in some denomination which has not a corresponding lineal measure, as acres or roods, reduce it first into one which has.

EXAMPLE 1. What is the area of a square whose side is 3 feet 10 inches?

$$\begin{array}{rrr} \text{ft.} & \text{in.} & \\ 3 & 10 & \\ 3 & 10 & \\ \hline 11 & 6 & \\ 3 & 2 & 4 \\ \hline 14 & 8 & 4 \end{array}$$

Ans. 14 feet, 8 parts, 4 inches.*

EXAMPLE 2. The area of a parallelogram is required, its length being 745 links, and its breadth 367.

$$\begin{array}{rl} \left.\begin{array}{r}745\\367\end{array}\right\} & \text{links.} \\ \hline 5215 & \\ 4470 & \\ 2235 & \\ \hline 100{,}000)273415 & \text{sq. links.} \\ \hline 2{\cdot}73415 & \text{acres.} \\ 4 & \\ \hline 2{\cdot}93660 & \text{roods.} \\ 40 & \\ \hline 37{\cdot}46400 & \text{poles.} \end{array}$$

Ans. 2 acres, 2 ro., 37½ – pls.

---

* In operations in duodecimals the square foot is usually divided into 12 equal parts called inches. But this is peculiarly inconvenient, since there are thus two kinds of superficial inches—one the twelfth of a foot, and the other the 144th part. Throughout this volume I adhere to the method of Professor Bonnycastle, dividing the square foot into twelve *parts*, and the part into twelve square inches.

EXAMPLE 4. Compute the length, in chains, of the side of a square whose area is 1 acre 3 roods.

1 ac. 3 ro.	280 (16·73 + pls.	
4	1	
7 ro. (See Note 3)	26)180	4)16·73
40	156	
		Ans. 4·18 + chains.
280 pls.	327)2400	
	2289	
	3343)11100	
	10029	
	1071	

EXERCISE 1. The side of a square measures 9·72 inches: what is its area?

Ans. 94·4784 sq. inches.

2. How many square yards are contained in a floor 46 feet by 29?

Ans. 148 yds. 2 feet.

3. The area of a rectangle is required, 7⅛ inches long, and 5⅓ inches broad.

Ans. 38 sq. inches.

4. The side of a rhombus is 19 ft. 6 in., and its altitude 12 ft. 8 in.: what is its area in sq. yards?

Ans. 27 yds. 4 ft.

5. A deal board is 26½ feet long, and 11¾ inches broad: how many square feet of surface does it present when laid in a floor? Ans. 25 ft. 11 pts. 4½ in.

6. What is the area of a rectangular field, the length of which is 944 links, and the breadth 587.

Ans. 5 ac. 2 ro. 6½ + pls.

7. The base of the largest Egyptian pyramid is a square, whose side is 745 feet: how many acres of ground does it cover, and how many cottages could stand close together on the same area, each 22 feet by 15?

First Ans. 12·742 – acres.
Second Ans. 1682 cottages.

8. At 1s. 2½d. per square yard, what will be the expense of paving a court 84 feet long and 47¾ feet wide?

Ans. £26 : 18 : 6¼ –.

9. What is the superficial area of a cube of 7⅝ inches?

Ans. 2 sq. feet 60$\frac{27}{32}$ sq. inches.

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10. How many square feet of deal will be sufficient to make a box 3 feet long, 2 ft. 6 in. broad, and 1 ft. 4 in. deep, outside measure, making no deduction for the corners.

Ans. 29 sq. ft. 96 in.

11. What is the area of the floor of the room in which you are, and also (separately from the floor) of the walls, omitting the door, windows, and fire-place?

18. What is the side of a square whose area is 4830¼ square inches?

Ans. 5 ft. 9½ in.

19. The area of a square is 70 ft. 10 pts. 1 in.: what is the length of the side?

Ans. 8 ft. 5 in.

20. What is the side of a square whose area is 5?

Ans. 2·23607 –.

21. Find the side of a square whose area is an acre.

Ans. 69·6 – yds.

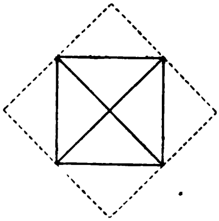
Problem II.,

To find the Area of a Square when the Diagonal is given.

Rule. Multiply the diagonal by itself, and halve the product.

FORMULA.

Area $= \frac{1}{2}D^2$.



REV. FORMULA

Diag. $= \sqrt{2A}$

Exercise 1. What is the area of a square whose diagonal is 24?

Ans. 288.

2. How many acres are in a square space whose diagonal measures 5 chains?

Ans. 1¼ acre.

3. The area of a square is 578 square yards: what is the diagonal?

Ans. 34 yards.

PROBLEM III.

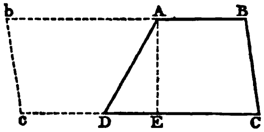
To find the Area of a Trapezoid (ABCD) *having given its two Parallel Sides* (AB *and* CD), *and its Breadth* (AE).

RULE. Multiply the sum of the two parallel sides by the breadth, and half the product will be the area.

FORMULA.

$$\text{Area} = \tfrac{1}{2}(S + S') \times B.$$

S and S' being the par. sides, and B the perpend. breadth.



NOTE. Instead of halving the product, we may halve the breadth, when an even number, or both the sides when even numbers, or the sum of the sides when both are odd.

EXERCISE 1. The two parallel sides of a trapezoid are 5 and 8, and its breadth 3: what is its area? Ans. $19\frac{1}{2}$.

2. Determine the area of a trapezoid having its parallel sides 24 and 18, and their perpendicular distance 27. Ans. 567.

3. The area of a trapezoid is required, the parallel sides being 17 ft. 6 in., and 14 ft. 10 in., and the perpendicular between them 13 ft. 7 in. Ans. 24 yds. 3 ft. $7\frac{1}{6}$ pts.

4. How many acres does a field contain, the four sides of which are 3·48, 2·99, 8·31, and 4·60 chains, the first side being parallel to the third, and 2·76 chains distant from it? Ans. 1 ac. 2 ro. $20\frac{1}{3}$ – pls.

PROBLEM IV.

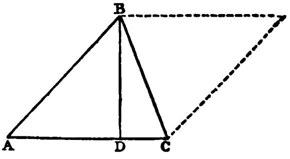
To find the Area of a Triangle (ABC), *having given its Base* (AC), *and Altitude* (BD).

RULE. Multiply the base by the altitude, and halve the product.

FORMULA.

$$\text{Area} = \frac{B \times P}{2}.$$

B being the base and P the perp. height or altitude.



Note. If the altitude or base is an even number, we may multiply the base by half the altitude, or the altitude by half the base.

Example. The area of a triangle is required, the base being 5·96; and the perpendicular, or altitude, 3·81 chains.

$$
\begin{array}{r l}
2)5{\cdot}96 & \text{chains.} \\
\hline
2{\cdot}98 & \\
3{\cdot}81 & \\
\hline
298 & \\
2384 & \\
894 & \\
\hline
10)11{\cdot}3538 & \text{sq. chains.} \\
\hline
1{\cdot}13538 & \text{acres.} \\
4 & \\
\hline
0{\cdot}54152 & \text{roods.} \\
40 & \\
\hline
21{\cdot}66080 & \text{poles.}
\end{array}
$$

Ans. 1 ac. 0 ro. 22 − pls.

Exercise 1. What is the area of a triangle, the base being 7·37, and the altitude 4·98 chains?

Ans. 1 ac. 3 ro. 13½ + pls.

2. What is the area of a right-angled triangle, the base being 29½, and the perpendicular 33⅔?

Ans. $496\frac{7}{12}$.

3. The three sides of a triangle are 4 ft. 4 in., 2 ft. 1 in., and 5 ft. 3 in.; and the perpendicular let fall on the longest side is 1 ft. 8 in. Calculate the area.

Ans. 4 ft. 4 pts. 6 in.

4. The hypothenuse of a right-angled triangle is 205, and the base 200. Find the area. Ans. 4500.

5. Compute the number of square yards in an isosceles triangle, each of the two equal sides measuring 47 ft. 8 in., and the base 36 ft. 8 in.[*] Ans. 89 sq. yds. 5⅔ feet.

6. The side of an equilateral triangle being 3·4, what is the area? Ans. 5·0056 +.

7. What is the area of a hexagon whose side is 1?

Ans. 2·598076 +.

---

[*] This and the following Exercises must be performed without the help of any succeeding problem.

8. What is the superficial area of a square pyramid, base and sides; each side of the base being 35 feet; and the perpendicular height 42 feet?* Ans. 4410 sq. ft.

PROBLEM V.

To find the Area of a Triangle when the Three Sides are given.

RULE. Add together the three sides, and halve the sum. From the half sum subtract each side separately. Multiply the half sum and the three remainders together. The square root of the product will be the area.

NOTE 1. The first part of the work may be proved, by observing if the sum of the three remainders is equal to the half sum of the three sides.

EXAMPLE. The three sides of a triangle are 13, 20, and 21: what is its area?

```
  13      27      27      27       27
  20      13      20      21       14
  21      —       —       —       ———
  —       14       7       6      378
2)54                                7
  —                               ———
  27                              2646
                                     6
           14)                   ————
            7} Three Rems.       15876(126 Ans.
            6)                   144
           —                     ————
           27 Proof.          246)1476
                                  1476
```

NOTE 2. This rule is much used in *Land Surveying*. For this purpose it would be very troublesome (the operation being tedious when the numbers consist of several figures), were it not much facilitated by the use of LOGARITHMS.† These are employed for the multiplication and extraction of the root, the previous part of the work being performed in exactly the same manner as when the logarithms are not used.

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* The scholar will easily accomplish this with a model before him. Sets of models are kept, prepared, by the Edinburgh publishers of this work,—one set for the Elementary Course, and another for the Complete Course.

† See the Author's Practical Treatise on Logarithms, &c., or Part III of this work, now in preparation.

Exercise 1. The area of a triangle is required, the three sides being 13, 14, and 15. Ans. 84.

2. If the three sides of a triangle are 2425, 2418, and 1481 links, how many acres of ground does it contain? Ans. 17 ac. 0 ro. 11½ ± pls.

3. What is the area of an isosceles triangle, the base measuring 2 ft. 11 in., and each side 4 ft. 3 in.?* Ans. 5 sq. ft. 118·3 + sq. in.

4. The three sides of a right-angled triangle are 20, 21, and 29. Compute its area by this rule and prove it by Problem IV. Ans. 210.

5. The side of an equilateral triangle being 7·96 feet, what is its area in acres? Ans. 6 ac. 1 ro. 7·7 + pls.

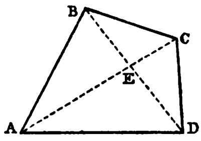
6. Required: the total superficial content of the base and sides of a triangular pyramid, each side of the base being 1·45 inch, and each slant edge 2·68 inches. Ans. 6·522 ± sq. inches.

Note. More Exercises in this rule will be found under Problems VI and VIII.

Problem VI.

To find the Area of any Quadrilateral Figure (ABCD).

Rule. Draw either diagonal, dividing the quadrilateral into two triangles, and find the areas of these separately (either by Pr. IV or by Pr. V). Their sum will be the area of the quadrilateral.



Exercise 1. In a trapezium ABCD, given one diagonal AC = 556 links; and the two perpendiculars let fall on AC from B and D, 264 and 235 links. The area is required. Ans. 1·38722 ac.

2. In another trapezium the four sides are as follows, AB = 628, BC = 464, CD = 457, DA = 733, and the diagonal AC = 835 links. Compute the area. Ans. 3·1108 acres.

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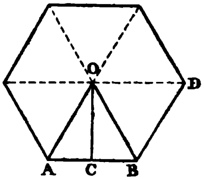
* In this Exercise the feet must first be reduced to inches.

PROBLEM VII.

To find the Area of any regular Polygon, of not more than twelve Sides, when the Side is given.

RULE. Multiply the square of the side by the number opposite the name of the Polygon in the column of areas in Table I.



FORMULA. Area $= L^2 \times a$.

L being the length of the side; and a the tabular area.

EXAMPLE. What is the area of a regular heptagon, the side being 7 yds. 2 ft.?

yds. ft. feet.	Tab. area $= 3{\cdot}6339$
7 . . 2 = 23	529
23	
69	32 7051
46	72 678
	1816 95
$L^2 = 529$.	9)1922·3331 sq. feet.

Ans. 213 sq. yds. $5\frac{1}{3} \pm$ sq. ft.

EXERCISE 1. The area of a pentagon is required, the side being 3·82. Ans. 25·106 –.

2. A part of a town is laid out in the form of an octagon, each side being 156 feet. How many acres does it contain? Ans. 2 ac. 2 ro. $31\frac{1}{2}$ + pls.

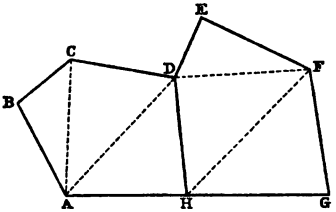
3. What is the area of an equilateral triangle, the side measuring 389 links? Ans. 2 ro. 24·84 – pls.

4. What is the area of the whole surface of a hexagonal prism, whose length is $8\frac{1}{2}$ inches, and each side of the hexagonal ends 1·43 inch? Ans. 33·556 – sq. inches.

5. Compute the area of the base and sides of a heptagonal pyramid, each side of the base being 3 ft. 9 in., and the slant height 5 ft. 4 in.? Ans. 121 sq. ft. 15 – sq. in.

PROBLEM VIII.

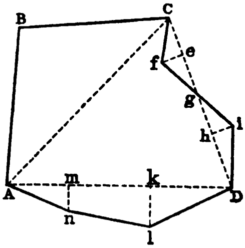
To find the Area of any Irregular Figure.



RULE. Divide the figure, as directed in Chap. II, Pr. LXXVI. Then calculate the areas of the separate triangles, by Problem IV or Problem V. When there are offsets (as in the figure accompanying the Example), the small spaces between them may be regarded as trapezoids and triangles, and their areas calculated by Problems III and IV.

NOTE. If the boundary is curved, no material error will arise from regarding the small portions between the offsets as straight, particularly if the places of the latter are so adjusted that the deficiencies in one part are compensated by excesses in another part.

EXAMPLE. Required: the area of a field similar to the annexed figure, the dimensions being as follows. AB = 331 links, BC = 312, CD = 384, AD = 473, AC = 489, Ce = 80, Cg = 163, Ch = 260, ef = 45, ih = 46, Am = 132, Ak = 304, mn = 51, kl = 81.



Triangle ABC=	51004	sq. links.
... ACD=	84602	
... giD=	5083	
... Dkl=	6844	
Trapezoid kmnl=	11352	
Triangle Amn=	3366	
	162251	
Deduct △ Cfg=	3667	
Field ABCD=	158584	sq. links.
	1·58584	acre.
	4	
	2·34336	roods.
	40	
	13·73440	pls.

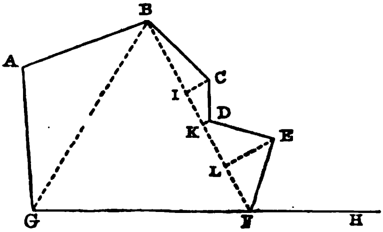
Ans. 1 ac. 2 ro. 13¾ pls., nearly.

EXERCISE 1. Required: the areas of the two fields represented in the first diagram of this Problem, the sides and diagonals being of the following lengths. AB = 10·42 chains, BC = 6·77, AC = 14·21, CD = 11·02, AD = 16·74, DH = 12·23, AH = 12·48, HF = 18·88, DF = 14·53, DE = 7·30, EF = 12·68, FG = 13·61, and GH = 14·76.

Ans. ABCDH = 18·682 −; DEFGH = 23·489 −. Total = 42·170 + acres.

2. Compute the area of the field sketched in the following plan, the dimensions being as annexed.

Links.
GF = 680
GB = 693
BF = 678
AG = 448
AB = 414
BI = 257
BK = 369
BL = 512
CI = 82
DK = 22
EL = 170.

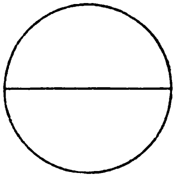


Ans. 3 ac. 1 ro. 16·4 − pls.

PROBLEM XII.

To find the Area of a Circle when the Radius and Circumference are given.

RULE. Multiply the circumference by the radius, and halve the product.



FORMULA.

$$\text{Area} = \frac{C \times R}{2}$$

EXERCISE 1. The diameter of a circle being 1 and its circumference 3·1416, what is its area?

Ans. ·7854.

2. The radius of a circle being 2 ft. 5½ in. and its circumference 15 ft. 5·35 in., what is its area?

Ans. 18 sq. feet 142 − sq. in.

Problem XIII.

To find the Area of a Circle when the Radius, or Diameter, only is given.

Rule. Multiply the square of the radius by 3·1416 –.

Formula. $\text{Area} = R^2 \times \Pi.$

Eaxmple. The diameter of a circle being 18, what is its area, true to four figures?

$$\begin{array}{lr} & 3{\cdot}1416 \\ \text{Rad.}^2 = 9^2 = & 81 \\ \hline & 3\,1416 \\ & 251\,328 \\ \hline \text{Ans. } 254{\cdot}5 - . & 254{\cdot}4696 \end{array}$$

Exercise 1. What is the area of a circle, the radius being 26? Ans. 2124 –.

2. Calculate the number of acres in a circular area whose radius is 379 links. Ans. 4 ac. 2 ro. 2·0 + pls.

3. The diameter of a circle being 25·84, what is its area? Ans. 524·4 +.

4. What is the area of a circular table, the diameter of which is 4 ft. 11 in.? Ans. 18 sq. ft. 142 – sq. in.

5. The diameter of a circle is $17\frac{4}{11}$. Find the area. Ans. $236\frac{8}{10}$ nearly.

Problem XIV.

To find the Area of a Circle when the Circumference only is given.

Rule 1. Divide one-fourth of the square of the circumference by 3·1416 –.

Formula. $\text{Area} = \frac{C^2}{4} \div \Pi.$

Rule 2. Multiply the square of the circumference by ·07958 –.

Formula. $\text{Area} = C^2 \times \frac{1}{4\,\Pi}.$

Exercise. Compute the area of a circle whose circumference is $9\frac{1}{5}$. Ans. 6·735 +.

PROBLEM XV.

To find the Area of a Sector of a Circle.

RULE. Find the length of the arc by any of the rules for that purpose in Part III. Multiply this by the radius, and halve the product.

FORMULA. $$\text{Area} = \frac{\text{Arc} \times \text{Rad.}}{2}$$

EXERCISE 1. The length of an arc is 27, and the radius of the circle 15. What is the area of the space included between the arc and the two radii drawn to its extremities? Ans. 202½.

2. I require the area of a sector of 107½ degrees, the length of the circumference of the whole circle being 78. Ans. 144·57.

3. How many square yards are contained in a sector of 93° 48½′, the diameter of the circle being 578 feet? Ans. 7597 sq. yds.

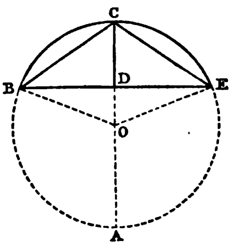
4. The chord of the arc of a sector is 254, and the chord of half the arc 153: what is the area of the sector, computing the arc's length by Problem XIII, Chapter III? Ans. 22 (177).

5. What is the area of a sector, the chord of its arc being 12, and the height of the arc 3, determining the length of the arc by Pr. XIII, Ch. III? Ans. 52·(082).

6. What is the area of the same sector, determining the length of the arc by Pr. XII, Ch. III? Ans. 52·16.

PROBLEM XVI.

To find the Area of a Segment of a Circle (BCE).



RULE 5. *If the number of degrees in the segment is given,* look for that number in the right-hand column of Degrees in Table II. *If the chord is given,* divide half the chord by the radius, and look for the quotient in the column of Nat. Sines. Or, *if the height of the segment is given,* divide it by the radius, and look for the quotient in the column headed N. V. Sine.

Having found any one of these numbers in the Table, take the number opposite in the column of segments, and multiply that number by the square of the radius.

Note. The answer, by this rule, will not be correct to more than the number of figures in the tabular segment.

If the segment is greater than a semicircle, find the segment BAE, which will be less than a semicircle, and subtract it from the area of the whole circle.

Rule 6. $$\text{Area} = \frac{4}{3} h \times \sqrt{\left(\frac{1}{4} c^2 + \frac{4}{10} h^2\right)} - .$$

h being the height and c the chord.

Example 1. What is the area of a segment of 35° 20′, the diameter of the circle being 80?

By Rule 5. Turning to Table II, and looking down the right-hand column of degrees, we find 35° 20′, and opposite this, in the column of Seg., ·0192. This, multiplied by R^2 or 1600, gives 30·7 for the segment.

Example 2. The chord of a segment is $3\frac{1}{5}$, and the radius 2. The area of the segment is required.

By Rule 5. Chord = 3·2. Half chord = 1·6. 1·6 ÷ 2 = ·8.

Looking for ·8 in the column of N. Sines in Table II, we find the nearest number ·8004, corresponding to the segment ·4481. This, multiplied by 4 (the square of the radius), gives 1·79(24) for the answer.

By Rule 6. $\text{OD} = \sqrt{(\text{BO}^2 - \text{BD}^2)} = 1{\cdot}2.$

$h = \text{CD} = \text{OC} - \text{OD} = {\cdot}8,$ and $h^2 = {\cdot}64.$

$$\begin{array}{lr}
\frac{1}{4}\,c^2 = 2{\cdot}56 & 1{\cdot}678094 \\
\frac{4}{10}\,h^2 = \ \ {\cdot}256 & {\cdot}8 = h \\
\hline
\sqrt{2{\cdot}816} = 1{\cdot}678094 + . & 3)1{\cdot}342475 \\
 & \overline{{\cdot}447492} \\
 & \overline{1{\cdot}789(967).\ \text{Ans.}}
\end{array}$$

EXERCISE 1. Find the area of a segment whose arc contains 90°, the radius of the circle being 24.
Ans. 164·39.

2. What is the area of a segment whose arc is a sextant, the whole circumference of the circle being 1, by Rule 5? Ans. ·00229.

3. The chord of a segment is 24, and the diameter of the whole circle 25. Compute the area of the segment by Rule 5. Ans. 159, nearly.

4. The height of an arc is 27, and the radius of the circle 37·5. Find the area of the segment by Rule 5.
Ans. 143(3)−.

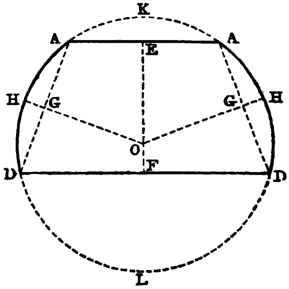
5. The chord of a segment is 20, and its height 4: what is its area, by Rule 5? Ans. 55·(2+).

6. Determine the same by Rule 6. Ans. 55·0(1+).

PROBLEM XVII.

To find the Area of a Circular Zone (AAHDDH).

RULE. The zone being divided into a trapezoid (AADD), and two equal segments (AHD and AHD), find the area of the trapezoid (by Pr. III), and the areas of the two segments (by Pr. XVI). These three areas, added together, will be the area of the zone.



EXERCISE 1. The two parallel chords of a circular zone are 18 and 24; and their distances from the centre, viz., OE and OF, 12 and 9. Find the area of the zone.
Ans. 569·43.

2. I require the area of a circular zone, each of its parallel chords being 15 inches, and the diameter of the circle 20 inches. Ans. 1 sq. ft. 100 sq. in., nearly.

3. The radius of a circle is 75. A zone of that circle has one of its parallel chords coinciding with the diameter, and the other equal to the radius. What is the area of the zone? Ans. 8326 +.

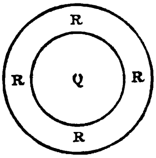
PROBLEM XVIII.

To find the Area of a Circular Ring (RRRR).

RULE. Find the areas of the two circles separately, and their difference will be the area of the ring.

FORMULA.

$$\text{Area} = (R^2 - r^2) \times \Pi, \text{ or}$$
$$\text{Area} = (C^2 - c^2) \times \frac{1}{4\Pi}, \text{ or}$$
$$\text{Area} = \tfrac{1}{2}(C + c) \times (R - r).$$



R and *r* being the radii of the two circles, and *C* and *c* their circumferences.

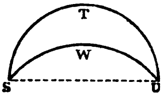
EXERCISE 1. What is the area of a circular ring, the diameters of the inner and outer circles being 8 and 10 feet respectively? Ans. 28·274 feet.

2. A circular shrubbery is surrounded by a road of uniform breadth, the inner side of the road measuring 357 yards in circumference, and the outer side 400 yards. What quantity of ground does the road cover? Ans. 2590 sq. yards.

PROBLEM XIX.

To find the Area of a Lune (STUW).

RULE. Find the areas of the two segments (STU and SWU) formed by the chord of the lune and the two arcs. The difference of these will be the area of the lune.



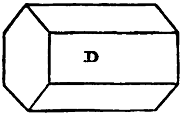
EXERCISE. The chord of a lune is 48, and the heights of the two arcs (STU and SWU) which bound it, 10 and 7. The area of the lune is required. Ans. 103 +.

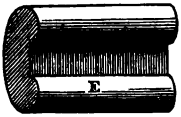
PROBLEM XX.

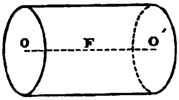
To find the Area of the Convex Surface of a right Cylinder, or the Area of the Sides of a right Prism.

RULE. Multiply the circumference by the length.

FORMULA. Area $= C \times L$.







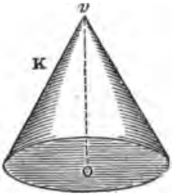
EXERCISE 1. The length of a cylinder is 2 ft. 5 in., and the circumference 13 inches. The area of its convex surface is demanded. Ans. 2 sq. ft. 89 sq. in.

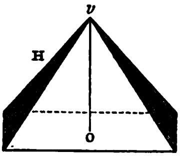
2. What is the *whole* superficial area of a right cylinder $7\frac{1}{2}$ feet long, and 2 ft. 8 in. in diameter? Ans. $8\frac{2}{9}$ sq. yds.

3. The length of a right hexagonal prism is 8·5, and each side of the hexagon 1·43. What is its *whole* surface? Ans. 83·556 –.

PROBLEM XXII.

To find the Area of the Convex Surface of a right Cone, or the Area of the Sides of a regular right Pyramid.





RULE. Multiply the circumference of the base by the slant height, and halve the product.

FORMULA. Area $= \dfrac{C \times S}{2}$

Note. By the slant height of a pyramid is meant—not the slant edge, but the altitude of any one of the triangular sides.

Exercise 1. The slant height of a cone is 44 feet, and the circumference of its base 27¼ feet. What is the convex surface? Ans. 599½ sq. feet.

2. How many square yards of painting will be required for an octagonal wooden spire 100 feet in slant height, and 54 ft. 5 in. in circumference at the base?

Ans. 302 yds. 3 ft., nearly.

3. The diameter of the base of a cone is 5·8, and its slant height 7·35. The convex surface is wanted.

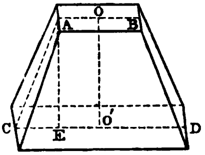
Ans. 66·96 +.

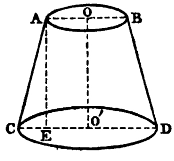
4. A conical tin vessel is close on all sides: the diameter of its base is 30 inches, and its perpendicular altitude 20 inches: how many square feet of tin does its *whole* surface present? Ans. 13 sq. ft. 13 in.

5. Each side of the hexagonal base of a pyramid is 10, and its perpendicular height 20. Determine the whole surface. Ans. 913·6 +.

PROBLEM XXIII.

To find the Convex Surface of the Frustum of a right Cone, or the Area of the Sides of the Frustum of a regular Right Pyramid.





Rule. Multiply the sum of the circumferences of the two ends by the slant height (AC) of the frustum, and halve the product.

Formula. $$\text{Area}=\frac{C+c}{2}\times s.$$

Or, *in the case of a cone,*

$$\text{Area} = \frac{D+d}{2} \times s \times \Pi. \qquad \text{Or, Area} = (R+r) \times s \times \Pi.$$

C and c being the circumferences, D and d the diameters, and R and r the radii of the two ends, and s the slant height of the frustum.

Exercise 1. The frustum of a cone is $3\frac{5}{8}$ inches in circumference at the base, and $2\frac{3}{4}$ at the top. Its slant height is $5\frac{3}{8}$. Find the convex surface.
Ans. 17·13 + sq. inches.

2. The frustum of a cone is 7·4 inches in slant height, and the diameters of its two ends are 8·3 and 6 inches. The convex surface is required. Ans. 166·22 sq. in.

3. The *perpendicular* altitude of the frustum of a cone is 1 foot, and the circumferences of the base and top 3 feet and 2 feet respectively. What is the *whole* surface?
Ans. 2·5315 –.

4. The slant height of the frustum of a hexagonal pyramid is 74 feet: each side of the base measures 15 feet, and each side of the top 5 feet. Compute the superficial area of the sides. Ans. 4440 sq. feet.

Problem XXVI.

To find the Superficial Area of a Sphere.

Rule 1. Multiply the circumference of the sphere by its diameter.

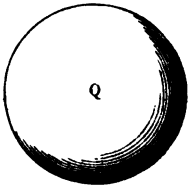
Rule 2. Multiply the square of the diameter by 3·1416 –.

Formulæ.

$$\text{Area} = C \times D.$$

$$\text{Area} = D^2 \times \Pi.$$

$$\text{Area} = C^2 \times \frac{1}{\Pi}.$$



Exercise 1. The radius of a sphere is 1, and the circumference 6·283. Compute the convex surface by Rule 1. Ans. 12·566.

2. What is the surface of a sphere whose diameter is 3? Ans. 28·274 +.

3. The surface of a sphere is required, the circumference being 1. Ans. ·31831.

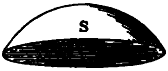
PROBLEM XXVII.

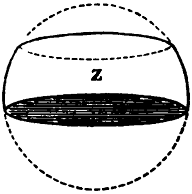
To find the Convex Surface of a Segment, or of a Zone, of a Sphere.

RULE. Multiply the circumference of the whole sphere by the height* of the segment or zone.

FORMULA.

$$A = C \times h.$$





EXERCISE 1. The radius of a sphere is 411·9. Find the convex surface of a segment whose height is 342·5. Ans. 886404.

2. The diameter of a sphere is 4 ft. 11 in. From the opposite sides of the sphere two segments are cut off by parallel sections, the heights of these segments being 12 and 14 inches respectively. What is the convex surface of the remainder of the sphere? Ans. 42 sq. ft. 69 – in.

PROBLEM XXX.

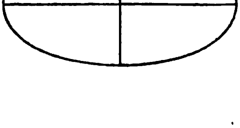
To find the Area of an Ellipse.

RULE. Multiply the product of the axes by ·7854 –, or the product of the semi-axes by 3·1416 –.

FORMULA.

$$\text{Area} = ab \times \Pi.$$

$$\text{Or, Area} = 2a \times 2b \times \frac{\Pi}{4}.$$



$2a$ and $2b$ being the two axes.

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* By the "height" of the zone, we mean what is elsewhere called the perpendicular breadth.

Exercse 1. What is the area of an ellipse whose axes are 3 and 2? Ans. 4·7124 –.

2. An elliptical plot of ground is 2·50 chains in length, and 1·48 in breadth: what is its area? Ans. 1 ro. $6\frac{1}{2}$ pls., nearly.

Problem L.

Having given two corresponding Lineal Dimensions of two similar Figures, and an Area in the one Figure, to find the corresponding Area in the other.

Rule. State a proportion, having, for its third term, the given area, and, for its first and second terms, the squares of the two given lineal dimensions, placing first that which is connected with the given area. The fourth term, when found, will be the area required.

Formula. $L^2 : l^2 :: A : a$.

L and l being the two corresponding lines, and A and a the two corresponding areas of the same figures.

Example. If the surface of a sphere 1 foot in circumference be ·31831 of a square foot, what will be the surface of a musket-ball $1\frac{1}{2}$ inch in circumference?

$$\left.\begin{array}{l}\text{As } 12^2 : (1\tfrac{1}{2})^2 \\ \text{or, As } 144 : \frac{9}{4}\end{array}\right\} :: \cdot 31831 \text{ sq. ft.} : \cdot 7162 \text{ sq. in. Ans.}$$

Exercise 1. If the area of a garden 125 yards in length be 2 ac. 0 ro. $23\frac{3}{4}$ pls., what will be the area of a similar garden 80 yds. in length? Ans. 3 ro. $20\frac{4}{5}$ pls.

2. If a circle 100 feet in radius be 31,416 sq. feet in area, what will be the area of a circle $17\frac{4}{11}$ feet in radius? Ans. 947·2 – sq. feet.

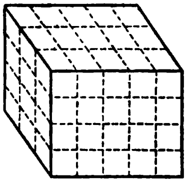
3. If the transverse axis of an elliptic piece of ground be 50 yards, and its area quarter of an acre, what will be the area of a similar ellipse whose transverse axis is 40 yards? Ans. 25·6 sq. pls.

4. If gilding a pyramidal spire for the first ten feet of slant height (commencing at the top) cost £5, 10s., what will be the cost of gilding the remainder at the same rate, the slant height of the whole spire being 72 feet? Ans. £279 : 12 : 5 –.

CHAPTER V.

MENSURATION OF SOLIDS.

A cube whose side is 1 inch in length, 1 foot, 1 yard, or other denomination, is called *a Cubic Inch*, *a Cubic Foot*, *a Cubic Yard*, or other denomination.



The *Solid Content* of a figure means the number of cubic inches, cubic feet, cubic yards, or other denomination, which it contains or is equal to.

Thus, the solid content of the figure represented in the diagram is 60 cubic inches, because it contains 60 of the small cubes, each 1 inch long, 1 inch broad, and 1 inch thick.* The solid content is sometimes called the *Solidity*.

TABLE OF SOLID MEASURES.

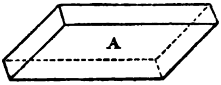
1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
$\left.\begin{array}{l}277{\cdot}274\text{, or}\\ 277\frac{1}{4}\text{, nearly}\end{array}\right\}$ cubic inches = 1 gallon.

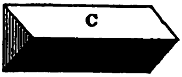
GENERAL NOTE.

Throughout all the Problems in Part v, the letter *S* is used to denote the solid content of the figure to which the problem applies.

Problem I.

To find the Solid Content of a Cube or other Parallelopiped, or of any Prism or Cylinder, Right or Oblique.





Rule. Multiply the area of the base by the perpendicular height.

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* The teacher may explain this by a model.

NOTE. One end of the prism or cylinder is here regarded as its base, and any one of the six sides of the parallelopiped.

FORMULA. $S = A \times H$.

For the Cube, $S = L^3$. *Reversely,* $L = \sqrt[3]{S}$.

For the Rectangular Parallelopiped, $S = L \times B \times H$.

A being the area of the base, and L, B, and H being the length, breadth, and height or thickness.

EXAMPLE 1. What is the solid content of a cube whose side is 3 ft. 9 in. in length?

By Common Arithmetic.

3 ft. 9 in.
12
45 inches.
45
225
180
2025 sq. in.
45
10125
8100
91125 ÷ 1728 =
52 c. ft. 1269 c. in.

By Decimals.

12)9·0
3·75
3·75
1875
2625
1125
14·0625
3·75
703 125
9843 75
42187 5
52·734 375 c. ft.

By Duodecimals.

3 ft. 9 in.
3 .. 9 ..
11 .. 3 ..
2 .. 9 .. 9
14 ft. 0 pa. 9 in.
3 .. 9
42 .. 2 .. 3
10 .. 6 .. 6 .. 9
52 c.f. 8 pa. 9 sec. 9 in.*

By Fractions.

3 ft. 9 in. $= 3\frac{3}{4}$ ft. $= \frac{15}{4}$.

$$\left(\frac{15}{4}\right)^3 = \frac{3375}{64} = 52\tfrac{47}{64}.$$

Ans. $52\frac{47}{64}$ cubic feet.

---

* In operations by duodecimals, in the following problems, the cubic foot is divided into 12 parts, a part into 12 seconds, and a second into 12 cubic inches.

Example 2. Find the solid content of a cylinder whose length is 8 and diameter 6.

$$\begin{array}{r l}
3{\cdot}1416 & \\
9 & = \text{Radius}^2. \\
\hline
28{\cdot}2744 & = \text{Area of Base.} \\
8 & \\
\hline
22{\cdot}619(52), & \text{Ans.}
\end{array}$$

Example 3. What is the length, breadth, or thickness of a cube whose solid content is 52 c. ft. 8 pts. 9 sec. 9 c. in.?

$$\begin{array}{l}
12)9{\cdot}00 \text{ in.} \\
\hline
12)9{\cdot}75 \text{ sec.} \\
\hline
12)8{\cdot}8125 \text{ pts.} \\
\hline
\quad 52{\cdot}734\,375 \text{ c. feet.} \\
\quad 52{\cdot}734,375\ (3{\cdot}75 \text{ feet, Ans.} \\
\quad 27 \\
\hline
\end{array}$$

$$\begin{array}{r c r l}
30^2 \times 3 & = & 2700) & 25{\cdot}734 \\
30 \times 7 \times 3 & = & 630 & \\
7^2 & = & 49 & \\
\hline
7 \times & & 3379 & = 23{\cdot}653 \\
\hline
370^2 \times 3 & = & 410700) & 2{\cdot}081\,375 \\
370 \times 5 \times 3 & = & 5550 & \\
5^2 & = & 25 & \\
\hline
5 \times & & 416275 & = 2{\cdot}081\,375
\end{array}$$

Exercise 1. What is the solid content of a cube, each of whose edges measures 1·25 ft.?

Ans. 1 c. ft. 1647 c. in.

2. The content of a cube is required, measuring 3 ft. 4 in. every way. Ans. 37 c. ft. 0 pa. 5 sec. 4 in.

This is more systematic than the common practice of calling the twelfth part of a cubic foot an inch, and thus having two kinds of solid inches altogether distinct. (See foot note to page 73.)

3. Messrs Irby and Mangles describe the stones used in the various edifices of Baalbec as immensely large. "We measured," they say, "a single stone of sixty-six feet in length, and twelve in breadth and thickness." What would be the content of such a stone, and what its weight, at the rate of 180 lb. to a cubic foot, the material being "coarse marble excessively hard?"

Ans. Content 9504 c. feet. Weight 764 tons, nearly.

4. A beam is 25 ft. 9 in. long, 1 ft. 5 in. broad, and 9 inches thick. How many cubic feet of timber does it contain? Ans. 27 ft. 4 pa. 3 s. 9 in.

5. What is the solid content of the room in which you are, in cubic yards, neglecting the smaller recesses?

6. A hexagonal prism is 16 inches in length, each side of the base measuring 3 inches. Compute its solidity.

Ans. 374·1 +.

7. Find the solid content of a right cylinder, its length being 3 ft. 6 in., and its diameter 1 ft. 3 in.

Ans. 4·295 + c. ft.

8. A prism is 6 inches in length, the sides of its triangular base being 2, 3, and 4 inches. The solid content is required.

Ans. 17·4284 + c. inches.

9. What is the length of the side of a cube of 15$\frac{5}{8}$ cubic feet?

Ans. 2$\frac{1}{2}$ feet.

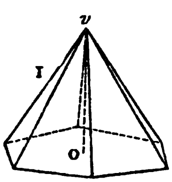
10. If the length of the side of a cube be 3, what will be the length of the side of a cube of twice its bulk?*

Ans. 3·78 –.

PROBLEM II.

To find the Solidity of a Cone or Pyramid (Right or Oblique).

RULE. Multiply the area of the base by the perpendicular height, and divide the product by 3.



FORMULA. $S = \frac{A \times H}{3}$

---

* This is the famous problem of doubling the cube proposed by the Oracle to the Athenians. It can only be resolved approximately either by Geometry or by Arithmetic.

Exercise 1. The solidity of a cone is required, the diameter of the base being 6, and the perpendicular height 8. Ans. 75·40 –.

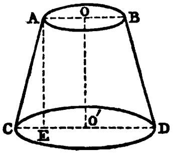
2. What is the solid content of a square pyramid, the side of the base being 460 feet, and the height 315 feet? Ans. 822,889 – c. yards.

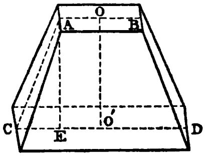
3. Compute the weight of a small hexagonal pyramid of marble, each side of the horizontal base measuring 2 ft. 6 in., the vertical height being 5 feet, and marble weighing about 170 lb. to the cubic foot. Ans. 2 tons 1 cwt 9 lb.

4. The circumference of the base of a right cone is 32·9867, and the slant height 8·75. Find the solid content. Ans. 202·04 +.

Problem III.

To find the Solid Content of a Frustum of a Cone or Pyramid (Right or Oblique).





Rule. To the areas of the two ends add the square root of their product. Multiply this sum by the height, and divide the product by 3.

Formula. $S=(E+e+\sqrt{Ee})\times\frac{1}{3}h.$

E and e being the areas of two ends, and h the perpendicular height.

Exercise 1. The areas of the two ends of a frustum of a cone are 4 and 3 sq. inches, and their perpendicular distance from each other is $1\frac{3}{4}$ inch. How many cubic inches are contained in the frustum? Ans. 6·104 +.

2. The frustum of a square pyramid has the sides of its base each $3\frac{1}{2}$, those of its top each $2\frac{1}{2}$, and its height 3. The solid content is required. Ans. $27\frac{1}{4}$.

3. The largest of the pyramids of Egypt is more strictly a frustum of a pyramid, having each side of its square base 745 feet, each side of its square summit 32 feet, and

its vertical height 480 feet. What is its solid content and its weight at the rate of 150 lb. to the cubic foot?

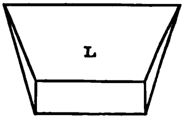
Ans. 3,436,379 cubic yds., and 6,213,096 tons.

4. The frustum of a cone has the diameters of its two ends 16 and 12, and its perpendicular height 5. Compute the solid content by the Rule. Ans. 775 — .

PROBLEM VII.

To find the Solid Content of a Wedge.

RULE. Add the length of the edge to twice the length of the base. Multiply the sum by the height of the wedge, and the product again by the breadth of the base. One-sixth of the last product will be the solidity.



FORMULA. $S=\frac{1}{6}(2L+L')\times H\times B.$

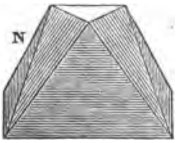
EXERCISE 1. The edge of a wedge is 4 ft. 6 in. long, the base 2 ft. 8 in. long and 1 ft. 4 in. broad, and the height (or perpendicular distance of the edge from the base) is 1 ft. 9 in. How many solid feet does it contain?

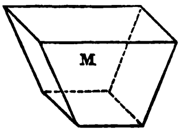
Ans. 3 c. ft. 9 pa. 10 se. 8 in.

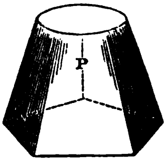
2. Another wedge has its height, the length of its edge, the length of its base, and the breadth of its base, severally, 1 inch. Its solidity is required. Ans. $\frac{1}{2}$ c. inch.

PROBLEM VIII.

To find the Solid Content of a Prismoid.



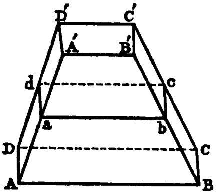




RULE. Find the area of a section parallel to the two ends, and equidistant from both. To four times that area add the areas of the two ends; and this last sum, multiplied by one-sixth of the length or height, will give the content.

Formula. $S = \frac{1}{6}(A + 4a + A') \times H.$

Note. If the ends (ABCD and A′B′C′D′) are rectangles, with their corresponding sides parallel, the middle section will also be a rectangle (abcd), having its length (ab) half the sum of the lengths of the two ends (AB and A′B′), and its breadth (ad) half the sum of the breadths of the two ends (AD and A′D′).



Example. What is the solid content of a prismoid whose ends are rectangles, having the sides of the one parallel to the corresponding sides of the other, the one rectangle being 10 feet by 4, and the other 8 by 6, and their perp. distance from each other 5?

$$\begin{array}{r} 10 = L. \\ 8 = L'. \\ \hline 2)18 \\ \hline 9 = l \end{array} \qquad \begin{array}{r} 4 = B. \\ 6 = B'. \\ \hline 2)10 \\ \hline 5 = b \end{array} \qquad \begin{array}{rr} l = & 9 \\ b = & 5 \\ \hline a = & 45 \\ & 4 \\ \hline 4a = & 180 \\ A = & 40 \\ A' = & 48 \\ \hline & 268 \\ H = & 5 \\ \hline & 6)1340 \\ \hline \text{Ans.} & 223\frac{1}{3}\text{ c. ft.} \end{array}$$

L, *B*, and *A* being put for the length, breadth, and area of the top; *L′*, *B′*, and *A′*, of the bottom; and *l*, *b*, and *a*, of the middle section; and *H* for the height.

Exercise. What is the solid content of a rectangular prismoid, whose greater end measures 16 in. by 7, and less end 10 in. by 5; the height being 2 feet?

Ans. 1 c. ft. 168 in.

PROBLEM XI.

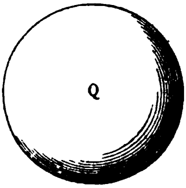
To find the Solid Content of a Sphere.

RULE 1. Multiply the cube of the diameter by ·5236 –.

FORMULA.

$$S = D^3 \times \frac{\Pi}{6}, \text{ or}$$

$$S = \frac{1}{6}(D^3 \times \Pi).$$



RULE 2. Multiply one-third of the surface by the radius.

EXERCISE 1. Compute the solid content of an ivory ball $1\frac{3}{4}$ inches in diameter, and its weight at the rate of 1820 ounces (Av.) to the cubic foot.

Ans. Content, 2·806 c. in. Weight, 3 ounces, nearly.

2. The solidity of a globe is required 3 feet 8 inches in diameter. Ans. 25 ft. 1402 + in.

3. What is the solid content of the sun, considering it as a sphere of 441,500 miles in radius; and how many times the size of the earth, reckoning it also a sphere of 3956 miles in radius?

Ans. About 360,480;000,000;000,000 cubic miles, or about 1;390,000 times the bulk of the earth.

PROBLEM XII.

To find the Solid Content of a Segment of a Sphere.

RULE 1. To three times the square of the radius of its base add the square of its height. Multiply the sum by the height, and the product again by ·5236 –.

FORMULA.

$$S = (3r^2 + H^2) \times H \times \frac{\Pi}{6}.$$

Rule 2. From three times the diameter of the sphere, subtract twice the height of the segment. Multiply the difference by the square of the height, and the product by ·5236 −.

Formula. $S=(3D-2H)\times H^2\times\frac{\Pi}{6}$.

Exercise 1. The height of a segment of a sphere is 2, and the diameter of its base 8. What is its solidity?

Ans. $54\frac{1}{2}$ −.

2. From a sphere, whose diameter is 6 feet 3 inches, a segment is cut off whose height is 2 feet 3 inches. Compute the solid content of the segment.

Ans. 37·77 + c. ft.

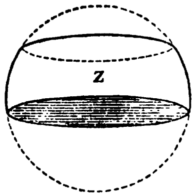
3. The diameter of a sphere is 33·8. What is the content of a segment less than a hemisphere, the radius of whose circular base is 12?

Ans. 1196 +.

Problem XIII.

To find the Solid Content of a Zone of a Sphere.

Rule. To three times the sum of the squares of the radii of the two ends add the square of the height of the zone. Multiply the sum by the said height, and the product again by ·5236 −.



Formula. $\mathrm{S}=\left\{3(\mathrm{r}^2+r^2)+h^2\right\}\times h\times\frac{\Pi}{6}$.

Exercise 1. The solid content of a zone of a sphere is required, the diameters of the ends being 40 and 30, and the breadth or height of the zone 20.

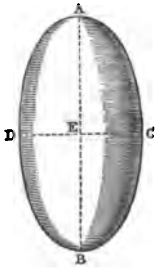
Ans. 23824 −.

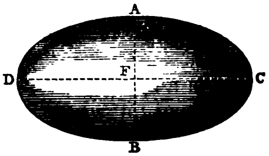
2. The radii of the two ends of a zone of a sphere are 52 and 60, the centre of the sphere being outside of the zone. The radius of the sphere itself is 65. What is the content of the zone?

Ans. 140,069 −.

PROBLEM XVIII.

To find the Solid Content of a Spheroid (ACBD).





RULE. Multiply the square of the revolving axis by the fixed axis and the product again by ·5236 –.

FORMULA. $S = R^2 \times F \times \frac{\Pi}{6}$.

R being the revolving axis, and *F* the fixed axis.*

EXERCISE 1. Compute the solid content of an oblate spheroid whose axes are 2 and 3. Ans. 9·425 –.

2. Compute the solid content of a prolate spheroid whose axes are 2 and 3. Ans. 6·283 +.

3. The Earth's bulk is required in cubic miles, the Polar axis being 7899 miles, and the Equatorial diameter 7925½ miles. Ans. 259,791 millions of cubic miles.

PROBLEM XLIII.

To find, approximately, the Content of any Solid, of nearly a Rectangular Form, when the Base and Top, Sides and Ends, are all plain Surfaces, or nearly so.

NOTE 1. This problem is used principally for mounds, heaps, and excavations.

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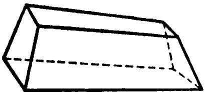
* It will be seen, from the definitions, that, in the prolate spheroid, the revolving axis is the conjugate, but, in the oblate, the revolving axis is the transverse.



Rule. Multiply the mean length by the mean breadth, and the product by the mean height or depth.

For the mean height, take the perpendicular height at each of the four corners; add the four heights together, and divide the sum by 4. *For the mean length*, take the lengths of the two sides at top and bottom; add the four lengths together, and divide the sum by 4. *For the mean breadth*, take the lengths of the two ends at top and bottom, and divide the sum of these also by 4.

Note 2. If it be asked whether the length and breadth are to be measured at right angles to each other, or in the oblique directions in which the sides and ends actually lie, the answer is, that, when the obliquity is very slight, it is immaterial which; since the oblique measurements will often give a more correct result than those taken perpendicularly, the content derived from the latter being always too small. When the obliquity is great, this rule cannot be applied with safety; but if it is used, the perpendicular dimensions are, in that case, best. The breadth should also be taken perpendicular to the length, when the horizontal surface (or section) assumes perceptibly the form of an oblique-angled parallelogram.

Note 3. This rule may be applied with safety when the slope of the sides is considerable, or even very great, if the ends are upright; or, when the slope of the ends is great, if the sides are upright; or, when the slope both of the ends and of the sides is great, if the height is *much* smaller than either the length or the breadth; and provided that, in all such cases, the *perpendicular* height be taken.

Example. It is required to determine the price of a dung heap at 5s. 6d. the cubic yard, the heap being nearly rectangular, and the sides, &c., straight, the heights at the four corners being 5 ft. 8 in., 5 ft. 10 in., 5 feet, and 5 ft. 2 in.; the lengths of the one side at top and bottom 25 feet, and 24 feet; of the other side 25 ft. 6 in., and 24 ft. 6 in.;

and the four breadths at the two ends 10 ft. 4 in., 9 ft. 6 in., 10 ft. 2 in., and 9 ft. 4 in.

Heights.	Lengths.	Breadths.
Ft. in.	Ft. in.	Ft. in.
5 .. 8	25 .. 0	10 .. 4
5 .. 10	24 .. 0	9 .. 6
5 .. 0	25 .. 6	10 .. 2
5 .. 2	24 .. 6	9 .. 4
4)21 .. 8	4)99 .. 0	4)39 .. 4
5 .. 5	24 .. 9	9 .. 10

5 .. 5			
9 .. 10			
48 .. 9	Ft.	Ft.	D.
4 .. 6 .. 2	27 :	1318¼ ::	66
	—	2	—
53 .. 3 .. 2 × 24¾	9		22
12		2636½	
		11	
639 .. 2 .. 0			
2		9)29001½	
1278 .. 4 .. 0		12)3222½	
26 .. 7 .. 7			
13 .. 3 .. 9½		20)268 .. 6½	
1318 .. 3 .. 4½		£13 .. 8 .. 6½ Ans.	

EXERCISE. Required: the expense of making an excavation having its base, sides, &c., flat surfaces, the four lengths taken being 51, 50, 49, and 48 feet; the four breadths 20 ft. 8 in., 20 ft. 4 in., 19 feet, 18 ft. 8 in.; and the four depths 11 ft. 8 in., 11 ft. 4 in., 10 ft. 4 in., and 10 feet, and the work being contracted for at the rate of 8½d. the solid yard.

Ans. £13 : 16 : 8.*

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* The scholar can be properly exercised in this Problem, only by actual measurements.

PROBLEM XLVII.

Having given two corresponding Lineal Dimensions of two similar Bodies, and the Solid Content of one of them (or of any Part of it), to find the Solid Content of the other (or of the corresponding Part of it).

RULE. State a proportion having for its third term the given solidity, and, for its first and second terms the cubes of the two lineal dimensions, placing first that which is connected with the given solidity. The fourth term, when found, will be the area required.

FORMULA. $L^3 : l^3 :: S : s$.

L and l being the two lineal dimensions, and S and s the two corresponding solids.

NOTE. When the two bodies are of the same substance, their weights may be used instead of their solid contents.

EXAMPLE 1. If a cannon ball, 8½ inches in diameter, weigh 6lb., what will be the weight of another of the same metal, 6·7 inches in diameter?

As $3{\cdot}5^3 : 6{\cdot}7^3 :: 6$ lb. That is,

As $42{\cdot}875 : 300{\cdot}763 :: 6 : 42 +$ lb. Ans.

EXAMPLE 2. A machine, 16 tons in weight, is 20 feet in length. A model of the same machine is to be constructed, of the same material, to weigh only 5 cwt. What length of box will be required to pack it in?

As 16 tons : 5 cwt. :: $20^3 : l^3$. Or,

320 cwt. : 5 cwt. :: $8000 : 125 = l^3$.

$\therefore l = \sqrt[3]{125} = 5$ feet, inside. Ans.

EXERCISE 1. If a cistern, 20 feet long, contain 6480 gallons, how many gallons will a similar cistern contain, 15 feet long? Ans. 2734 − gal.

2. If a man, 5 feet 6 inches in height, weigh 15 stones, what will be the weight of a man similarly proportioned, 6 feet high? Ans. 19½ stones, nearly.

3. If a model of a steam engine weigh 72 lb., what will be the weight of the engine itself, made of the same materials as the model, and of eight times its lineal dimensions? Ans. 16½ tons nearly.

CHAPTER VI.

PROMISCUOUS EXERCISES.

EXERCISES.

1. The sides of three squares being 3, 4, and 12, it is required to find the side of a square that shall be equal in area to all the three. Ans. 13.

2. How many leaden balls of a quarter of an inch in diameter can be cast out of the metal of a ball 3 inches in diameter, supposing no waste? Ans. 1728.

3. In cutting the largest possible circle out of a piece of card-board ten inches square, how many square inches must necessarily be wasted? Ans. 21·46 +.

4. In cutting four equal circles, the largest possible, out of the same square, how much would be wasted?
Ans. 21·46 + sq. inches.

5. The area of a square is 18 square inches, what is its diagonal? Ans. 6.

6. The solid content of a cube is 125, what is its whole superficial area? Ans. 150 sq. inches.

7. The three sides of a triangle were 800, 500, and 1237 links. By some mistake the third side was also put down as 500 instead of 1237. What error would that mistake occasion in the computed area?

Ans. No error at all worth estimating, being less than ·006 of a square chain.

CHAPTER VII.

UNRESOLVED EXERCISES.

EXERCISES IN CHAPTER III, PROBLEM I.

1. Base = 56. Perp. = 33. The hyp. is required.
2. Hyp. = 445. Base = 400. The perp. is required.

3. Hyp. = 7 ft. 5 in. Perp. = 3 ft. 3 in. The base is required.

4. The side of a square is 2 : what is the diagonal?

5. The diagonal of a square is 2 : what is the side?

6. The side of a square is $7\frac{1}{2}$ feet. What will be the distance from one corner to the middle of one of the opposite sides?

7. A footpath goes up the side, and then along the end, of a rectangular field, 216 yards long and 195 broad. What distance will be saved by cutting right across in the direction of the diagonal?

8. A ladder, 65 feet long, placed with its foot 33 feet from a wall, reaches within 7 feet of the top. How near the wall must the foot of the ladder be brought that it may reach the top?

EXERCISES IN CHAPTER III, PROBLEM VI.

1. Given, diameter = 26. The circumf. is required.
2. Diam. = $18\frac{3}{4}$. Circumf. required.
3. Radius = 7 ft. 4 in. Circumf. required.
4. Circumf. = 2. Diameter required.
5. Circumf. = 73·68. Radius required.
6. Circumf. = $9\frac{5}{12}$. Diameter required.

EXERCISES IN CHAPTER III, PROBLEM VII.

1. Height = 2·8. Chord = 6·3. The diameter is wanted.

2. The height of an arc is 17, and the chord four times as much. What is the radius?

3. Height = 42. Chord = 270. Diameter required.

EXERCISES IN CHAPTER III, PROBLEM VIII.

1. Height = 0·65. Chord of half arc = 3·9. Diameter required.

2. Height = $7\frac{3}{4}$. Chord of half arc = $15\frac{1}{2}$. Radius demanded.

3. Radius = 100. Height = 10. The chord of half the arc is asked for.

4. Diameter = 5·4. Chord of half arc = 2·4. The height of the arc is wanted.

EXERCISES IN CHAPTER III, PROBLEM IX.

1. Circumf. = 171. What is the length of an arc of 80°?

2. Circumf. = 54·28. What is the length of an arc of 154° 12′?

EXERCISES IN CHAPTER III, PROBLEM X.

1. Diameter = 2·5. Required the length of an arc of 76°.

2. Radius = 3¾. Compute the length of an arc of 98° 45′.

EXERCISES IN CHAPTER III, PROBLEM XII.

Find the respective arcs from the following data :—

1. Radius = 32. Sine = 28.
2. Diameter = 5½. Versed sine = 1½.
3. Diameter = 4·6. Chord = 3·9.
4. Radius = 37·24. Height of arc = 12·81.
5. Height = 196. Chord = 1344.
6. Radius = 20. Sine = 5⅓. The arc greater than a quadrant.
7. Height = 12. Chord = 18.

EXERCISES IN CHAPTER III, PROBLEM XIII.

Compute the respective arcs determined by the following dimensions :—

1. Chord of arc = 18. Chord of half arc = 15.
2. Chord = 5·6. Radius = 3·2.
3. Diameter = 11. Height = 3.
4. Height = 196. Chord = 1344.
5. Radius = 20. Chord = 10⅔. Arc greater than a semicircle.

EXERCISE IN CHAPTER III, PROBLEM XIX.

Transverse axis = 5. Conjugate axis = 4. The circumference is required by both rules.

EXERCISES IN CHAPTER III, PROBLEM XXVI.

1. If the direct distance from London to Edinburgh be 332 miles, what will be the distance in inches measured on a map laid down on a scale of 2 inches to 5 miles?

2. In a circle whose radius is 1, the chord of an arc of 90° is 1·4142 +. What will be the chord of an arc of 90° when the radius is $5\frac{1}{2}$?

3. On the preceding data, what will be the length of the chord of 60°, on a scale of chords whose extreme length is 6 inches, the scale extending, as usual, only as far as 90°?

4. A public building is 300 feet long, 180 feet wide, and 125 feet high. A model of it is to be made only 12 inches in length: what will be the width and height of the building as measured on the model?

5. A gentleman is 5 feet 8 inches in height, and his hat is 2 feet in circumference. Another is 6 feet in height, and his hat is 25 inches in circumference. Supposing their heads similarly formed, which of the two has the larger in proportion to his size?

6. If the radius of a circle, circumscribed about a pentagon, is 85065, when that of the inscribed circle is 68819, what will be the area of the circumscribed circle when that of the inscribed circle is 5·4?

EXERCISES IN CHAPTER IV, PROBLEM I.

1. The side of a square plot of ground is 25·4 feet. What is its area?

2. How many square yards are contained in a floor $35\frac{1}{2}$ feet by $22\frac{1}{2}$?

3. The longer side of an oblique-angled parallelogram is $5\frac{1}{3}$ inches, and its perpendicular breadth $2\frac{1}{4}$ inches. Calculate its area.

4. A large sheet of paper is 78 feet in length, and 3 ft. 8 in. wide. What is the area of the surface on one side, and, taking both sides, how many pages of paper will it be equivalent to, each 9 inches by $5\frac{1}{2}$?

5. A garden is 45 yards in length and 36 in width. What fraction of an acre is it?

6. Compute the entire superficial area of a rectangular block of freestone, 12 feet long, $5\frac{1}{2}$ feet wide, and 2 feet 4 inches thick.
7. At 9d. per square yard for the walls, and 1s. for the ceiling, what will be the expense of painting a room 54 feet long, 28 broad, and 15 feet 9 inches high?
8. The length of a rectangle is 24 inches, and its diagonal 26 inches. Find its area.
9. Determine the side of a square whose area is 8·4 sq. inches.
10. What must be the length of a square garden to contain just a quarter of an acre?
11. What must be the lineal dimensions of a cube, in order that its total superficial area may be just 1 square foot?

EXERCISE IN CHAPTER IV, PROBLEM II.

1. Find the area of a square whose diagonal is 5·956.
2. The diagonal of a square being 4 ft. 4 in., what is its area?

EXERCISES IN CHAPTER IV, PROBLEM III.

1. The two parallel sides of a trapezoid are 7·4 and 8·6, and its perpendicular breadth 4·5. Its area is required.
2. The two parallel sides measuring respectively $4\frac{3}{4}$ and $9\frac{5}{8}$, and the breadth $2\frac{3}{8}$ inches, what is the area?
3. The two parallel sides being 90 and 118, the one end being perpendicular to these, and the other end 100, compute the area from these data.

EXERCISES IN CHAPTER IV, PROBLEM IV.

The area of each of the following triangles is required from the data given :—

1. Base = 7·54. Altitude = 3·85.
2. Base = $18\frac{5}{8}$. Altitude = $3\frac{3}{8}$.
3. Base = 17 ft. 4 in. Altitude = 8 ft. 5 in.

4. In a right-angled triangle, hyp. = 125, base = 44.

5. One side of a triangle measuring 456 links, and the perpendicular upon that side from the opposite angular point measuring 351 links, what is the area in acres, roods, and poles, and what is the area in square yards?

6. The side of an octagon being 3, and the radius of its inscribed circle 3·621, calculate its area, without referring to any subsequent problem.

Exercises in Chapter IV, Problem V.

1. The three sides of a triangle are 26, 40, and 42. What is the area?

2. The three sides measuring respectively 12·56, 10·49, and 9·58 chains, what is the area of the triangle in acres?

3. The side of an equilateral triangle is 5 ft. 10 in. Find its area by the rule for this problem.

4. The three sides of a triangle being $4\frac{1}{4}$, $4\frac{1}{2}$, and $4\frac{3}{4}$, what is the area?

Exercises in Chapter IV, Problem VI.

1. In a four-sided field, ABCD, we have given the shorter diagonal, BD, 3·69 chains, and the perpendiculars let fall upon it from the points A and C, 4·76 and 3·58 chains. Its area is desired in acres, roods, and poles; and also, separately, in square yards.

2. In another four-sided field, the four sides measure respectively as follows,—AB = 745, BC = 582, CD = 486, and AD = 673; and the diagonal BD, 498. What is its area in acres and decimals of an acre?

Exercises in Chapter IV, Problem VII.

1. What is the area of a decagon whose side is 8·756?

2. What is the area of an equilateral triangle whose side is 5 ft. 10 in.

3. Calculate, by this rule, the whole surface of a dodecahedron, that is, of a solid having 12 faces, all regular pentagons, each edge of the solid (which is the same as each side of the pentagon) being $2\frac{3}{4}$ inches.

EXERCISES IN CHAPTER IV, PROBLEM VIII.

1. What would have been the separate and united areas of the two fields delineated in the first diagram under Problem VIII, if the different lines had been of the following measurements—AB = 521 links, BC = $338\frac{1}{2}$, AC = $710\frac{1}{2}$, CD = 551, AD = 837, DH = $611\frac{1}{2}$, AH = 624, HF = 944, DF = $726\frac{1}{2}$, DE = 365, EF = 634, FG = $680\frac{1}{2}$, and GH = 738?

2. What would have been the area of the field represented in the second diagram of Problem VIII, if the dimensions had been as follows, viz., AB = 9·93 chains, BC = 9·36, CD = 11·52, AD = 14·19, AC = 14·67, Ce = 2·40, Cg = 4·89, Ch = 7·80, ef = 1·35, ih = 1·38, Am = 3·96, Ak = 9·12, mn = 1·53, kl = 243?

EXERCISES IN CHAPTER IV, PROBLEM XII.

1. The radius of a circle being ·125, and its circumference ·7854, what is its area?

2. The radius being $7\frac{3}{8}$, and the circumference $46\frac{27}{80}$, the area is demanded.

EXERCISES IN CHAPTER IV, PROBLEM XIII.

1. What is the area of a circle whose diameter is $14\frac{3}{4}$?

2. Find the area of a circle whose radius is 2·35 chains.

3. What quantity of pasture can be grazed by a cow whose tether is 40 yards long, allowing the knots of the tether to compensate for the length the cow can stretch beyond it?

EXERCISES IN CHAPTER IV, PROBLEM XIV.

1. Given the circumf. = 15·94. The area is required.

2. What is the area (in acres or parts of an acre) of a circular enclosure, which measures 10 chains round?

EXERCISES IN CHAPTER IV, PROBLEM XV.

1. The length of an arc is 268·6, and the radius with which it is described 89·1. What is the area of the corresponding sector?

2. The radius of a circle being 2·588, what is the area of a sector of 45°?

3. Compute the area of a sector of 66° 30′, the whole circumference being 3·1416.

4. Find the area of a sector, having given its radius 32, and the sine of its arc 28.

5. The chord of an arc of a sector is 336, and the chord of half the arc 175. What is the area of the sector, finding the arc by Problem XIII of Chapter III?

6. What is the area of the same sector, computing the length of the arc by Pr. XII of Ch. III?

7. Ascertain the area of a sector, the chord of whose arc is 5·4, the radius of the circle being 6·9, and the length of the arc being computed by Pr. XII of Ch. III.

8. The chord of an arc is 13·44, and its height 1·96. Find the area of the space enclosed between that arc and the two radii drawn to its extremities, after determining the length of the arc by Pr. XII of Ch. III.

EXERCISES IN CHAPTER IV, PROBLEM XVI.

1. Required the area of a segment of 60°, the radius being 28.

2. Find the area of a segment of 26° 40′, the diameter of the circle being 9.

3. The chord of a segment is 67·2, and the diameter of the whole circle 125. Compute the area by Rule 5.

4. The same by Rule 6.

5. The diameter of a circle is 64, and the height of a segment 21. What is the area of the latter?

6. The chord of a segment is 84, and its height $17\frac{1}{2}$. Determine the area by Rule 5.

7. The same by Rule 6.

EXERCISES IN CHAPTER IV, PROBLEM XVII.

1. Find the area of a circular zone, the two parallel chords of which, on opposite sides of the centre, are 120 and 104, and their distances from the centre 25 and 39.

2. The two parallel chords being the same as in the last exercise, and at the same central distances, but both on the same side of the centre, compute the area of the zone.

3. What is the area of a circular zone having each of its parallel chords 2·5, and the diameter of the circle 4·8?

4. The two parallel chords of a circular zone are 75 and 100, the latter being the diameter of the circle. The area of the zone is demanded.

EXERCISES IN CHAPTER IV, PROBLEM XVIII.

1. Calculate the area of a circular ring whose width is 2·5, the radius of the outer circle being 12·75.

2. A circular place in a city is 100 yards in diameter. A paved road of 65 feet in width goes round it, enclosing a shrubbery in the centre, and leaving a distance of 5 yards between the road and the fronts of the houses, all round. What will be the cost of paving the said road, at 1s. 6d. per square yard?

EXERCISE IN CHAPTER IV, PROBLEM XIX.

1. The chord of a lune is 14·4, and the heights of the two arcs which bound it, 3 and 2·1. The area of the lune is required.

2. The diameters of two circles which cut each other are 10·2 and 6. The distance of the two points of intersection from each other is 4·8. What are the areas of the two lunes enclosed between the two circles?

EXERCISES IN CHAPTER IV, PROBLEM XX.

1. The length of a triangular prism is 4·36, and each side of its triangular ends 0·45. What is its whole superficial area?

2. The length of a cylinder being 2·5 and the radius of its ends the same, what is its *whole* surface?

3. The diameter of a garden roller is 2 ft. 10., and its length 3 ft. 2 in. How many times must it revolve before it roll over an acre of ground?

EXERCISES IN CHAPTER IV, PROBLEM XXII.

1. The slant height of a cone is 15·4 inches, and the circumference of its base 18·3 inches. Find the convex surface.

2. What is the *whole* surface of a cone, the radius of whose base is 5, and altitude 12?

3. Out of a sheet of tin-plate, 20 inches by 10, is to be cut a piece to form a conical vessel, 10 inches in slant height, and 6 inches in diameter at the base, which is to be left open. What fractional part of the tin-plate will be wasted?

4. A pyramid has a base of 10 feet square, and an altitude of 12 feet. What is the area of its four sides?

EXERCISES IN CHAPTER IV, PROBLEM XXIII.

1. The circumf. of the base of a conical frustum is 28·2, and that of the top 18·6, the slant height being 18·5. What is the convex surface?

2. Each side of the square base of the largest of the Egyptian pyramids (which is rather the frustum of a pyramid) is 745 feet, each side of the flat summit 32 feet, while the slant height (that is, the distance from the middle

of one side of the square summit to that of the corresponding side of the square base) is 598 feet. Of how many acres' extent are its sides and summit together?

3. A hexagonal pyramid of six feet in slant height, and having each side of its base 1·5 of a foot, is cut through at one-fourth of its height from the base. What is the united area of the six sides of the lower portion?

4. The frustum of a cone is 36 inches diameter at the base, and 24 at the top, and 8 inches in perpendicular height. Ascertain its convex surface.

Exercises in Chapter IV, Problem XXVI.

1. The diameter of a sphere is 4·5, and its circumference 14·137 nearly. What is the area of its surface by Rule 1?

2. The diameter of a sphere is $2\frac{1}{2}$. What is its surface?

3. Find the superficial area of a sphere whose radius is 19·9.

4. The circumf. of a sphere is 2·546. What is the area of the convex surface of the hemisphere?

Exercises in Chapter IV, Problem XXVII.

1. The diameter of a sphere being 14 feet 4 inches, compute the convex surface of a segment whose height is 2 feet 2 inches.

2. What is the convex surface of a zone of a sphere, the distances of the two ends from the centre being 2 and 3, and the diameter of the sphere 12?

3. The base of a spherical segment is 24 inches in diameter, and the height of the segment is 9 inches. What is the whole surface of the segment including the base?

4. A globe of 1 foot diameter has first a segment cut from it of $2\frac{1}{2}$ inches in height, and then another section is made parallel to the base of the segment, and at 4 inches distance from it. What are the respective areas of the convex surfaces of the segment first cut off,—of the zone,—and of the segment left?

EXERCISES IN CHAPTER IV, PROBLEM XXX.

1. What is the area of an elliptical enclosure whose length is 345 links and breadth half as much?

2. Calculate the area of the ring contained between the circumferences of two concentric ellipses, the one having its axes 6 and 3½, and the other 5½ and 3.

EXERCISES IN CHAPTER IV, PROBLEM L.

1. If it take 3½ yards of broad cloth to make a suit of clothes for a man 5 ft. 10 in. in height, how much will it take for a boy 4 ft. 2 in. in height, dressed in the same fashion, and proportional to the man in his dimensions?

2. If a balloon, 7 feet in height by 5 in width, requires 150 yards of silk to cover it, how much will be required for a similar balloon 15 ft. 5 in. in height?

3. A building is 362 feet in length, and covers 1·246 of an acre of ground. What area in square inches will be covered by a model of the building 50 inches in length?

4. The extreme length of the island of Great Britain is 530 miles, and its area is 67,160 square miles. How many square feet of surface will be contained in the island, as delineated on a map on which the extreme length, measured between the same two points, is 22 ft. 1 in.?

EXERCISES IN CHAPTER V, PROBLEM I.

1. What is the solid content of a cubical block of sandstone, measuring 2 ft. 8 in. every way?

2. How many cubic feet of timber are in a beam 30 ft. 6 in. long, 1 ft. 10 in. broad, and 1 ft. 2 in. thick?

3. What is the weight of a rectangular block of granite 48 feet long, 10 broad, and 8 thick, counting 175 lb. to the cubic foot?

4. What is the solid content of a right cylinder, whose diameter is 8·75, and length 26·24?

5. What is the solid content of a prism, whose length is 6·44 inches, each side of the hexagonal end measuring ·96 of an inch?

6. The solid content of a cube is 84·26. What is its length, breadth, or thickness?

7. If each edge of a cube measures 3 inches, what will be the length of the edge of another of half that bulk?

EXERCISES IN CHAPTER V, PROBLEM II.

1. What is the solid content of a cone whose base measures exactly 1 foot round, and whose vertex stands 15 inches above the base?

2. How many cubic feet are contained in an octagonal pyramid whose perp. height is 5·82 inches, each side of the base measuring 2·46 inches?

3. The slant height of a cone is 3·9, and the perp. height 3·6. What is the solid content?

EXERCISES IN CHAPTER V, PROBLEM III.

1. The frustum of a hexagonal pyramid has the sides of the summit and base 2 and 3, and the perp. distance between them $1\frac{1}{2}$. What is its content?

2. The slant height of a frustum of a cone is 10·6, and the perp. height 9. The diameter of the smaller end is 10. What is the solid content?

EXERCISE IN CHAPTER V, PROBLEM VII.

Calculate the solidity of a wedge having its base a rectangle 85·4 in length and 26·9 in breadth, its edge longer than its base by 10, and its height equal to twice the breadth of its base.

EXERCISE IN CHAPTER V, PROBLEM VIII.

How many cubic yards of soft clay will a cart contain, filled level with the brim, the bottom measuring 4 ft. 9 in.

by 3 ft. 3 in., the top 5 ft. 3 in. by 4 ft. 3 in., and the depth 1 ft. 1 in., inside? Compute also the weight of the clay, at the rate of one ton and a quarter to the cubic yard.

Exercises in Chapter V, Problem XI.

1. What is the solid content of the moon, taking it as a sphere of 2160 miles in diameter?

2. What is the weight of a cast-iron ball of 6 inches diameter, at the rate of 4·15 oz. Av. to the cubic inch?

3. What is the weight of a hemisphere of marble whose diameter is 2½ feet, reckoning 178 lb. to the cubic foot?

Exercises in Chapter V, Problem XII.

1. Calculate the solid measurement of a segment of a sphere, whose height is 7½, the radius of its base being 12¾.

2. The diameter of a sphere being 64·6, find the solid content of a segment whose height is 17·4.

3. The diameter of the base of a segment is 3·9, and the diameter of the sphere from which it is cut 8·9. What is the content of the segment?

Exercises in Chapter V, Problem XIII.

1. Calculate the solid content of a zone of a sphere, the diameters of its ends being 3·8 and 2·8, and the height of the zone 1·5.

2. The two ends of a zone of a sphere are on opposite sides of its centre, the one distant 3 ft. 3 in. from the centre and the other 1 ft. 4 in. The diameter of the sphere is 10 ft. 10 in. What is the solid content of the segment?

Exercises in Chapter V, Problem XVIII.

1. It is required to ascertain the solid content of a prolate spheroid having its axes 3 and 4.

2. The same of an oblate spheroid, of axes equal to the preceding.

EXERCISE IN CHAPTER V, PROBLEM XLIII.

What must be paid to a labourer for breaking a heap of stones for a road, at 1s. 10d. per cubic yard, the heap being of no regular form, but having its sides flat surfaces, and its upper surface and base nearly rectangular; the four lengths taken being 25 ft. 1 in., 24 ft. 10 in., 30 ft. 6 in., and 30 ft. 7 in.; the four breadths, 6 ft. 6 in., 6 ft. 8 in., 11 ft. 6 in., and 11 ft. 8 in.; and the four heights, 3 ft. 3 in., 3 ft. 2 in., 3 feet, and 3 ft. 4 in.?

EXERCISES IN CHAPTER V, PROBLEM XLVII.

1. If a hen's egg, 6·2 inches in its longest circumference, weigh 2 oz. Av., what should an ostrich's egg weigh, 17·9 inches in circumference, supposing it of similar form and of the same specific gravity?

2. If a hewn stone, 6 feet long, 3 feet broad, and 1 ft. 6 in. thick, weigh 2 tons 3 cwt., what will a stone of similar shape weigh 5 ft. 4 in. long?

3. If the Earth be regarded as a sphere of 7912½ miles in diameter, and the Moon as also a sphere of 2160 miles in diameter, how many times the bulk of the Moon is the Earth?

4. If the girt of an ox be 6 ft. 6 in., its live weight 88 stones, and the weight of its four quarters 52 st. 11 lb., what will be the live weight, and the weight of the four quarters, of an ox of like proportion, whose girt is 5 ft. 9 in.?

5. The height of a wooden pyramid is 15 inches, and its weight 3½ lb. The pyramid is sawn right across at one-third of its height above the base. What are the weights of the two portions?

EXERCISES IN CHAPTER VI.

1. Each side of a rhombus is 8·65, and its shorter diagonal 10·38. What is its longer diagonal?

2. When you attain the height of 5 feet 10 inches, what

length will you require your looking-glass to be, in order to see yourself from head to foot?

3. Three poles stand upright, in a straight line with each other, and on level ground. The heights of the two extreme poles are 31 and 25 feet: the top of the former is 20 feet, and that of the latter 26 feet, from the top of the middle pole; while the middle pole itself stands 12 feet distant from the higher of the other two. What is the distance between the tops of the two extreme poles?

4. To ascertain the weight of an acre of turnips, a drill was cleared of them to the length of ten yards, and the weight of those pulled from that portion was found to be 108 pounds. The drills were 28 inches apart. What was the weight of the turnips per acre, supposing the portion weighed to be a fair specimen?

5. The great building for the exhibition of the industry of all nations during the present year (1851) is 1851 feet in length, 408 feet in width, and 66 feet in height.* It covers 18† acres of ground, and contains 900,000 square feet of glass. Its total contents are 33 millions of cubic feet. If a model of the building is constructed 20 feet in length, what should be its width and height; how many square feet of floor will it cover; how many square feet of glass will be required for it, and what will be its cubic content?

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* That is, according to the plan approved by her Majesty's Commissioners previously to the erection of the building. Some additions have been made subsequently.

† Taking in, of course, a little more than the rectangular space described above.

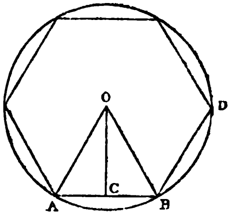
CHAPTER VIII.

TABLES.

TABLE I.

OF POLYGONS.

For the uses of this Table, see Pr. LI, Pt. II, and Note; also Pr. VII, Pt. IV, Rule II.



No. of Sides.	Name.	Angle AOB	Angle OAB	Angle ABD	Area.
3	Trigon*	120°	30°	60°	0·43301 +
4	Tetragon†	90°	45°	90°	1·0000
5	Pentagon	72°	54°	108°	1·7205 –
6	Hexagon	60°	60°	120°	2·5981 –
7	Heptagon	$51\frac{3}{7}°$	$64\frac{2}{7}°$	$128\frac{4}{7}°$	3·6339 +
8	Octagon	45°	$67\frac{1}{2}°$	135°	4·8284 +
9	Nonagon	40°	70°	140°	6·1818 +
10	Decagon	36°	72°	144°	7·6942 +
11	Undecagon	$32\frac{8}{11}°$	$73\frac{7}{11}°$	$147\frac{3}{11}°$	9·3656 +
12	Dodecagon	30°	75°	150°	11·1962 –

* Or Equilateral Triangle.

† Or Square.

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
0° 0′	·00000	·00000	·000000	·000000	0° 0′
10′	291	291	04	0	20′
20′	582	582	17	0	40′
30′	873	873	38	0	1° 0′
40′	·01164	·01164	68	1	20′
50′	454	454	·000106	2	40′
1° 0′	745	745	152	4	2° 0′
10′	·02036	·02036	207	6	20′
20′	327	327	271	8	40′
30′	618	618	343	·000012	3° 0′
40′	908	908	423	16	20′
50′	·03200	·03199	512	22	40′
2° 0′	491	490	609	28	4° 0′
10′	781	781	715	36	20′
20′	·04072	·04071	829	45	40′
30′	363	362	952	55	5° 0′
40′	654	653	·001083	67	20′
50′	945	943	222	81	40′
3° 0′	·05236	·05234	370	96	6° 0′
10′	527	524	527	·000112	20′
20′	818	814	692	131	40′
30′	·06108	·06105	865	152	7° 0′
40′	400	395	·002047	175	20′
50′	690	685	237	199	40′
4° 0′	981	976	436	227	8° 0′
10′	·07272	·07266	643	256	20′
20′	563	556	859	288	40′
30′	854	846	·003083	323	9° 0′
40′	·08145	·08136	315	360	20′
50′	436	426	556	400	40′
5° 0′	727	716	805	442	10° 0′
10′	·09018	·09005	·004063	488	20′
20′	309	295	329	537	40′
30′	599	585	604	589	11° 0′
40′	890	874	887	644	20′
50′	·10181	·10164	·005178	702	40′
6° 0′	472	453	478	764	12° 0′
10′	763	742	786	829	20′
20′	·11054	·11031	·006103	898	40′
30′	345	320	428	971	13° 0′
40′	636	609	762	·001047	20′
50′	926	898	·007103	128	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
21° 0′	·3665	·3584	·06642	·0320	42° 0′
10′	94	·3611	747	27	20′
20′	·3723	38	852	35	40′
30′	52	65	958	42	43° 0′
40′	82	92	·07065	50	20′
50′	·3811	·3719	173	58	40′
22° 0′	40	46	282	66	44° 0′
10′	69	73	391	75	20′
20′	98	·3800	501	83	40′
30′	·3927	27	612	91	45° 0′
40′	56	54	724	·0400	20′
50′	85	81	836	09	40′
23° 0′	·4014	·3907	950	18	46° 0′
10′	43	34	·08064	27	20′
20′	72	61	178	36	40′
30′	·4102	87	294	45	47° 0′
40′	31	·4014	410	54	20′
50′	60	41	528	64	40′
24° 0′	89	67	645	73	48° 0′
10′	·4218	94	764	83	20′
20′	47	·4120	884	93	40′
30′	76	47	·09004	·0503	49° 0′
40′	·4305	73	125	13	20′
50′	34	·4200	247	23	40′
25° 0′	63	26	369	33	50° 0′
10′	92	53	493	44	20′
20′	·4422	79	617	54	40′
30′	51	·4305	741	65	51° 0′
40′	80	31	867	76	20′
50′	·4509	58	993	87	40′
26° 0′	38	84	·10121	98	52° 0′
10′	67	·4410	248	·0609	20′
20′	96	36	377	20	40′
30′	·4625	62	507	32	53° 0′
40′	54	88	637	44	20′
50′	83	·4514	768	55	40′
27° 0′	·4712	40	899	67	54° 0′
10′	41	66	·11032	79	20′
20′	71	92	165	92	40′
30′	·4800	·4617	299	·0704	55° 0′
40′	29	43	434	16	20′
50′	58	69	569	29	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
28° 0′	·4887	·4695	·1171	·0742	56° 0′
10′	·4916	·4720	84	55	20′
20′	45	46	98	68	40′
30′	74	72	·1212	81	57° 0′
40′	·5003	97	26	94	20′
50′	32	·4823	40	·0808	40′
29° 0′	61	48	54	21	58° 0′
10′	91	74	68	35	20′
20′	·5120	99	82	49	40′
30′	49	·4924	96	63	59° 0′
40′	78	50	·1311	77	20′
50′	·5207	75	25	91	40′
30° 0′	36	·5000	40	·0906	60° 0′
10′	65	25	54	20	20′
20′	94	50	69	35	40′
30′	·5323	75	84	50	61° 0′
40′	52	·5100	99	65	20′
50′	81	25	·1413	80	40′
31° 0′	·5411	50	28	96	62° 0′
10′	40	75	43	·1011	20′
20′	69	·5200	58	27	40′
30′	98	25	74	43	63° 0′
40′	·5527	50	89	59	20′
50′	56	75	·1504	75	40′
32° 0′	85	99	20	91	64° 0′
10′	·5614	·5324	35	·1107	20′
20′	43	48	50	24	40′
30′	72	73	66	41	65° 0′
40′	·5701	98	82	58	20′
50′	30	·5422	97	75	40′
33° 0′	60	46	·1613	92	66° 0′
10′	89	71	29	·1209	20′
20′	·5818	95	45	27	40′
30′	47	·5519	61	44	67° 0′
40′	76	44	77	62	20′
50′	·5905	68	93	80	40′
34° 0′	34	92	·1710	98	68° 0′
10′	63	·5616	26	·1316	20′
20′	92	40	42	35	40′
30′	·6021	64	59	53	69° 0′
40′	50	88	75	72	20′
50′	80	·5712	92	91	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
21° 0′	·3665	·3584	·06642	·0320	42° 0′
10′	94	·3611	747	27	20′
20′	·3723	38	852	35	40′
30′	52	65	958	42	43° 0′
40′	82	92	·07065	50	20′
50′	·3811	·3719	173	58	40′
22° 0′	40	46	282	66	44° 0′
10′	69	73	391	75	20′
20′	98	·3800	501	83	40′
30′	·3927	27	612	91	45° 0′
40′	56	54	724	·0400	20′
50′	85	81	836	09	40′
23° 0′	·4014	·3907	950	18	46° 0′
10′	43	34	·08064	27	20′
20′	72	61	178	36	40′
30′	·4102	87	294	45	47° 0′
40′	31	·4014	410	54	20′
50′	60	41	528	64	40′
24° 0′	89	67	645	73	48° 0′
10′	·4218	94	764	83	20′
20′	47	·4120	884	93	40′
30′	76	47	·09004	·0503	49° 0′
40′	·4305	73	125	13	20′
50′	34	·4200	247	23	40′
25° 0′	63	26	369	33	50° 0′
10′	92	53	493	44	20′
20′	·4422	79	617	54	40′
30′	51	·4305	741	65	51° 0′
40′	80	31	867	76	20′
50′	·4509	58	993	87	40′
26° 0′	38	84	·10121	98	52° 0′
10′	67	·4410	248	·0609	20′
20′	96	36	377	20	40′
30′	·4625	62	507	32	53° 0′
40′	54	88	637	44	20′
50′	83	·4514	768	55	40′
27° 0′	·4712	40	899	67	54° 0′
10′	41	66	·11032	79	20′
20′	71	92	165	92	40′
30′	·4800	·4617	299	·0704	55° 0′
40′	29	43	434	16	20′
50′	58	69	569	29	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
42° 0′	·7330	·6691	·2569	·2358	84° 0′
10′	59	·6713	88	84	20′
20′	89	34	·2608	·2410	40′
30′	·7418	56	27	37	85° 0′
40′	47	77	47	63	20′
50′	76	99	67	90	40′
43° 0′	·7505	·6820	86	·2517	86° 0′
10′	34	41	·2706	44	20′
20′	63	62	26	72	40′
30′	92	84	46	99	87° 0′
40′	·7621	·6905	66	·2627	20′
50′	50	26	86	55	40′
44° 0′	79	47	·2807	82	88° 0′
10′	·7709	67	27	·2711	20′
20′	38	88	47	39	40′
30′	67	·7009	67	67	89° 0′
40′	96	30	88	96	20′
50′	·7825	50	·2908	·2825	40′
45° 0′	54	71	29	54	90° 0′
10′	83	92	50	83	20′
20′	·7912	·7112	70	·2913	40′
30′	41	33	91	42	91° 0′
40′	70	53	·3012	72	20′
50′	99	73	33	·3002	40′
46° 0′	·8029	93	53	32	92° 0′
10′	58	·7214	74	62	20′
20′	87	34	95	92	40′
30′	·8116	54	·3116	·3123	93° 0′
40′	45	74	38	53	20′
50′	72	94	59	84	40′
47° 0′	·8203	·7314	80	·3215	94° 0′
10′	32	33	·3201	46	20′
20′	61	53	23	78	40′
30′	90	73	44	·3309	95° 0′
40′	·8319	92	66	41	20′
50′	48	·7412	87	73	40′
48° 0′	78	31	·3309	·3405	96° 0′
10′	·8407	51	30	37	20′
20′	36	70	52	70	40′
30′	65	90	74	·3502	97° 0′
40′	94	·7509	96	35	20′
50′	·8523	28	·3417	68	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
35° 0′	·6109	·5736	·1808	·1410	70° 0′
10′	38	60	25	29	20′
20′	67	83	42	49	40′
30′	96	·5807	59	68	71° 0′
40′	·6226	31	76	88	20′
50′	54	54	93	·1508	40′
36° 0′	83	78	·1910	28	72° 0′
10′	·6312	·5901	27	48	20′
20′	41	25	44	68	40′
30′	70	48	61	89	73° 0′
40′	·6400	72	79	·1610	20′
50′	29	95	96	30	40′
37° 0′	58	·6018	·2014	51	74° 0′
10′	87	41	31	73	20′
20′	·6516	65	49	94	40′
30′	45	88	66	·1715	75° 0′
40′	74	·6111	84	37	20′
50′	·6603	34	·2102	59	40′
38° 0′	32	57	20	81	76° 0′
10′	61	80	38	·1803	20′
20′	90	·6202	56	25	40′
30′	·6720	25	74	48	77° 0′
40′	49	48	92	70	20′
50′	78	71	·2210	93	40′
39° 0′	·6807	93	29	·1916	78° 0′
10′	36	·6316	47	39	20′
20′	65	38	65	62	40′
30′	94	61	84	86	79° 0′
40′	·6923	83	·2302	·2010	20′
50′	52	·6406	21	33	40′
40° 0′	81	28	40	57	80° 0′
10′	·7010	50	58	81	20′
20′	39	72	77	·2106	40′
30′	69	94	96	30	81° 0′
40′	98	·6517	·2415	55	20′
50′	·7127	39	34	80	40′
41° 0′	56	61	53	·2205	82° 0′
10′	85	83	72	30	20′
20′	·7214	·6604	91	55	40′
30′	43	26	·2510	80	83° 0′
40′	72	48	30	·2306	20′
50′	·7301	70	49	32	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
42° 0′	·7330	·6691	·2569	·2358	84° 0′
10′	59	·6713	88	84	20′
20′	89	34	·2608	·2410	40′
30′	·7418	56	27	37	85° 0′
40′	47	77	47	63	20′
50′	76	99	67	90	40′
43° 0′	·7505	·6820	86	·2517	86° 0′
10′	34	41	·2706	44	20′
20′	63	62	26	72	40′
30′	92	84	46	99	87° 0′
40′	·7621	·6905	66	·2627	20′
50′	50	26	86	55	40′
44° 0′	79	47	·2807	82	88° 0′
10′	·7709	67	27	·2711	20′
20′	38	88	47	39	40′
30′	67	·7009	67	67	89° 0′
40′	96	30	88	96	20′
50′	·7825	50	·2908	·2825	40′
45° 0′	54	71	29	54	90° 0′
10′	83	92	50	83	20′
20′	·7912	·7112	70	·2913	40′
30′	41	33	91	42	91° 0′
40′	70	53	·3012	72	20′
50′	99	73	33	·3002	40′
46° 0′	·8029	93	53	32	92° 0′
10′	58	·7214	74	62	20′
20′	87	34	95	92	40′
30′	·8116	54	·3116	·3123	93° 0′
40′	45	74	38	53	20′
50′	72	94	59	84	40′
47° 0′	·8203	·7314	80	·3215	94° 0′
10′	32	33	·3201	46	20′
20′	61	53	23	78	40′
30′	90	73	44	·3309	95° 0′
40′	·8319	92	66	41	20′
50′	48	·7412	87	73	40′
48° 0′	78	31	·3309	·3405	96° 0′
10′	·8407	51	30	37	20′
20′	36	70	52	70	40′
30′	65	90	74	·3502	97° 0′
40′	94	·7509	96	35	20′
50′	·8523	28	·3417	68	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
49° 0'	·8552	·7547	·3439	·3601	98° 0'
10'	81	66	61	34	20'
20'	·8610	85	83	67	40'
30'	39	·7604	·3506	·3701	99° 0'
40'	68	23	28	35	20'
50'	98	42	50	69	40'
50° 0'	·8727	60	72	·3803	100° 0'
10'	56	79	94	37	20'
20'	85	98	·3617	71	40'
30'	·8814	·7716	39	·3906	101° 0'
40'	43	35	62	40	20'
50'	72	53	84	75	40'
51° 0'	·8901	71	·3707	·4010	102° 0'
10'	30	90	29	46	20'
20'	59	·7808	52	81	40'
30'	88	26	75	·4117	103° 0'
40'	·9018	44	98	52	20'
50'	47	62	·3820	88	40'
52° 0'	76	80	43	·4224	104° 0'
10'	·9105	98	66	60	20'
20'	34	·7916	89	97	40'
30'	63	34	·3912	·4333	105° 0'
40'	92	51	35	70	20'
50'	·9221	69	59	·4407	40'
53° 0'	50	86	82	44	106° 0'
10'	79	·8004	·4005	81	20'
20'	·9308	21	28	·4518	40'
30'	38	39	52	56	107° 0'
40'	67	56	75	94	20'
50'	96	73	99	·4631	40'
54° 0'	·9425	90	·4122	69	108° 0'
10'	54	·8107	46	·4708	20'
20'	83	24	69	46	40'
30'	·9512	41	93	84	109° 0'
40'	41	58	·4217	·4823	20'
50'	70	75	40	62	40'
55° 0'	99	92	64	·4901	110° 0'
10'	·9628	·8208	88	40	20'
20'	57	25	·4312	79	40'
30'	87	41	36	·5019	111° 0'
40'	·9716	58	60	58	20'
50'	45	74	84	98	40'

Degrees.		Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.	
56°	0′	·9774	·8290	·4408	·5138	112°	0′
	10′	·9803	·8307	32	78		20′
	20′	32	23	56	·5218		40′
	30′	61	39	81	59	113°	0′
	40′	90	55	·4505	99		20′
	50′	·9919	71	29	·5340		40′
57°	0′	48	87	54	81	114°	0′
	10′	77	·8403	78	·5422		20′
	20′	1·0007	18	·4602	63		40′
	30′	36	34	27	·5504	115°	0′
	40′	65	50	52	46		20′
	50′	94	65	79	87		40′
58°	0′	1·0123	80	·4701	·5629	116°	0′
	10′	52	96	25	71		20′
	20′	81	·8511	50	·5713		40′
	30′	1·0210	26	75	55	117°	0′
	40′	39	42	·4800	98		20′
	50′	68	57	25	·5840		40′
59°	0′	97	72	50	83	118°	0′
	10′	1·0327	87	75	·5926		20′
	20′	56	·8601	·4900	68		40′
	30′	85	16	25	·6012	119°	0′
	40′	1·0414	31	50	55		20′
	50′	43	46	75	98		40′
60°	0′	72	60	·5000	·6142	120°	0′
	10′	1·0501	75	25	86		20′
	20′	30	89	50	·6229		40′
	30′	59	·8704	76	73	121°	0′
	40′	88	18	·5101	·6318		20′
	50′	1·0617	32	26	62		40′
61°	0′	47	46	52	·6406	122°	0′
	10′	76	60	77	51		20′
	20′	1·0705	74	·5203	96		40′
	30′	34	88	28	·6540	123°	0′
	40′	63	·8802	54	85		20′
	50′	92	16	80	·6631		40′
62°	0′	1·0821	29	·5305	76	124°	0′
	10′	50	43	31	·6721		20′
	20′	79	57	57	67		40′
	30′	1·0908	70	83	·6813	125°	0′
	40′	37	84	·5408	58		20′
	50′	66	97	34	·6904		40′

TABLE II.

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
63° 0′	1·0996	·8910	·5460	·6950	126° 0′
10′	1·1025	23	86	97	20′
20′	54	36	·5512	·7043	40′
30′	83	49	38	90	127° 0′
40′	1·1112	62	64	·7136	20′
50′	41	75	90	83	40′
64° 0′	70	88	·5616	·7230	128° 0′
10′	99	·9001	42	77	20′
20′	1·1228	13	69	·7324	40′
30′	57	26	95	72	129° 0′
40′	86	38	·5721	·7419	20′
50′	1·1316	51	47	67	40′
65° 0′	45	63	74	·7514	130° 0′
10′	74	75	·5800	62	20′
20′	1·1403	88	27	·7610	40′
30′	32	·9100	53	58	131° 0′
40′	61	12	80	·7707	20′
50′	90	24	·5906	55	40′
66° 0′	1·1519	35	33	·7803	132° 0′
10′	48	47	59	52	20′
20′	77	59	86	·7901	40′
30′	1·1606	71	·6013	50	133° 0′
40′	36	82	39	99	20′
50′	65	94	66	·8048	40′
67° 0′	94	·9205	93	97	134° 0′
10′	1·1723	16	·6119	·8146	20′
20′	52	28	46	96	40′
30′	81	39	73	·8245	135° 0′
40′	1·1810	50	·6200	95	20′
50′	39	61	27	·8345	40′
68° 0′	68	72	54	95	136° 0′
10′	97	83	81	·8445	20′
20′	1·1926	93	·6308	95	40′
30′	56	·9304	35	·8546	137° 0′
40′	85	15	62	96	20′
50′	1·2014	25	89	·8646	40′
69° 0′	43	36	·6416	97	138° 0′
10′	72	46	43	·8748	20′
20′	1·2101	56	71	99	40′
30′	30	67	98	·8850	139° 0′
40′	59	77	·6525	·8901	20′
50′	88	87	52	52	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
70° 0′	1·2217	·9397	·6580	·9003	140° 0′
10′	46	·9407	·6607	055	20′
20′	75	17	35	106	40′
30′	1·2305	26	62	158	141° 0′
40′	34	36	89	210	20′
50′	63	46	·6717	262	40′
71° 0′	92	55	44	314	142° 0′
10′	1·2421	65	72	366	20′
20′	50	74	99	418	40′
30′	79	83	·6827	470	143° 0′
40′	1·2508	92	55	522	20′
50′	37	·9502	82	575	40′
72° 0′	66	11	·6910	627	144° 0′
10′	95	20	38	680	20′
20′	1·2625	28	65	733	40′
30′	54	37	93	786	145° 0′
40′	83	46	·7021	839	20′
50′	1·2712	55	48	892	40′
73° 0′	41	63	76	945	146° 0′
10′	70	72	·7104	998	20′
20′	99	80	32	1·0052	40′
30′	1·2828	88	60	105	147° 0′
40′	57	96	88	159	20′
50′	86	·9605	·7216	212	40′
74° 0′	1·2915	13	44	266	148° 0′
10′	45	21	72	320	20′
20′	74	28	·7300	374	40′
30′	1·3003	36	28	428	149° 0′
40′	32	44	56	482	20′
50′	61	52	84	536	40′
75° 0′	90	59	·7412	590	150° 0′
10′	1·3119	67	40	644	20′
20′	48	74	68	699	40′
30′	77	81	96	753	151° 0′
40′	1·3206	89	·7524	808	20′
50′	35	96	53	862	40′
76° 0′	65	·9703	81	917	152° 0′
10′	94	10	·7609	972	20′
20′	1·3323	17	37	1·1027	40′
30′	52	24	66	082	153° 0′
40′	81	30	94	137	20′
50′	1·3410	37	·7722	192	40′

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
77° 0'	1·3439	·9744	·7750	1·1247	154° 0'
10'	68	50	79	302	20'
20'	97	57	87	358	40'
30'	1·3526	63	·7836	413	155° 0'
40'	55	69	64	469	20'
50'	84	75	92	524	40'
78° 0'	1·3614	81	·7921	580	156° 0'
10'	43	87	49	636	20'
20'	72	93	78	691	40'
30'	1·3701	99	·8006	747	157° 0'
40'	30	·9805	35	803	20'
50'	59	11	63	859	40'
79° 0'	88	16	92	915	158° 0'
10'	1·3817	22	·8120	971	20'
20'	46	27	49	1·2027	40'
30'	75	33	78	084	159° 0'
40'	1·3904	38	·8206	140	20'
50'	34	43	35	196	40'
80° 0'	63	48	64	253	160° 0'
10'	92	53	92	309	20'
20'	1·4021	58	·8321	365	40'
30'	50	63	40	422	161° 0'
40'	79	68	78	479	20'
50'	1·4108	72	·8407	535	40'
81° 0'	37	77	36	592	162° 0'
10'	66	81	64	649	20'
20'	95	86	93	706	40'
30'	1·4224	90	·8522	763	163° 0'
40'	54	94	51	820	20'
50'	83	99	79	876	40'
82° 0'	1·4312	·9903	·8608	934	164° 0'
10'	41	07	37	991	20'
20'	70	11	66	1·3048	40'
30'	99	14	95	105	165° 0'
40'	1·4428	18	·8724	162	20'
50'	57	22	52	219	40'
83° 0'	86	25	81	277	166° 0'
10'	1·4515	29	·8810	334	20'
20'	44	32	39	391	40'
30'	73	36	68	449	167° 0'
40'	1·4603	39	97	506	20'
50'	32	42	·8926	564	40'

Degrees.	Arc.	N. Sine.	N. V. Sine.	Segment.	Degrees.
84° 0′	1·4661	·9945	·8955	1·3621	168° 0′
10′	90	48	84	679	20′
20′	1·4719	51	·9013	736	40′
30′	48	54	42	794	169° 0′
40′	77	57	71	852	20′
50′	1·4806	59	99	909	40′
85° 0′	35	62	·9128	967	170° 0′
10′	64	64	57	1·4025	20′
20′	93	67	86	083	40′
30′	1·4923	69	·9215	140	171° 0′
40′	52	71	44	198	20′
50′	81	74	73	256	40′
86° 0′	1·5010	76	·9302	314	172° 0′
10′	39	78	31	372	20′
20′	68	80	60	430	40′
30′	97	81	90	488	173° 0′
40′	1·5126	83	·9419	546	20′
50′	55	85	48	604	40′
87° 0′	84	86	77	662	174° 0′
10′	1·5213	88	·9506	720	20′
20′	43	89	35	778	40′
30′	72	90	64	836	175° 0′
40′	1·5301	92	93	894	20′
50′	30	93	·9622	952	40′
88° 0′	59	94	51	1·5010	176° 0′
10′	88	95	80	068	20′
20′	1·5417	96	·9709	126	40′
30′	46	97	38	184	177° 0′
40′	75	97	67	243	20′
50′	1·5504	98	96	301	40′
89° 0′	33	98	·9825	359	178° 0′
10′	63	99	55	417	20′
20′	92	99	84	475	40′
30′	1·5621	1·0000	·9913	533	179° 0′
40′	50	00	42	592	20′
50′	79	00	71	650	40′
90° 0′	1·5708	00	1·0000	708	180° 0′

ERRATUM.

Page 90. Answer to Ex. 3. For 2·5315—, read 3·565±.

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